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TRANSACTIONS OF THE I•R•E® PGAP IS A QUARTERLY PUBLICATION DEVOTED TO EXPERIMENTAL AND THEORETICAL PAPERS ON ANTENNAS AND WIRELESS PROPAGATION OF ELECTROMAGNETIC WAVES

MANUSCRIPTS should be submitted to John B. Smyth, Editor, U. S. Navy Electronics Laboratory, San Diego, California. Manuscripts should be original typewritten copy, double spaced, plus one carbon copy. References should appear as footnotes and include author's name, title, journal, volume, initial and final page numbers, and date. Each paper must have an abstract of not more than 200 words. News items concerning PGAP members and group activities should be sent to the News Editor, Mr. H. A. Finke, Polytechnic Research and Development Company, 55 Johnson Street, Brooklyn, New York.

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Application has been made for second class entry, at the post office at Menasha, Wisconsin, under the act of March 3, 1879. Acceptance for mailing at a special rate of postage is provided for in the act of February 28, 1925, embodied in Paragraph 4, Section 412, P. L. & R., authorized October 26, 1927.

news and views

AFTER the mailing of the first issue of the PGAP TRANSACTIONS the editors were a little like the parents of a child prodigy giving a concert—aware of their offspring's shortcomings, yet proud of the talent and potentials displayed. How does one measure readership reaction? The questionnaire approach was taken; return reply cards were sent to the membership and comments requested.

A gratifyingly large number of replies were received. Typical of the comments was that of J. E. Eaton who wrote, "I am very favorably impressed with the first issue of the TRANSACTIONS—with the format and evident editorial care, and with the level of the papers published. If you can maintain these standards, we shall have a distinguished professional journal."

Harold Wheeler's proposal that the PGAP be split into a PGA and PGP elicited some heated pros and cons.

Here are some of the comments on the *con* side:

G. H. Koether, Jr. of Glen Burnie, Md. wrote: "Before the Antenna & Propagation Group is subdivided, I think that we should have all arguments published so that the membership can decide. I wonder if the idea may be due to a small group which is looking only to its own welfare. I think subdivision and specialization can very easily be carried too far. A large part of the membership is not specialized and benefits from a broad scope. Also, subdivision can defeat its own end by increasing costs to the point where group membership will drop off and the group die out. United we stand—divided we fall. Let us have no sudden moves in any direction.

"Technical publications are the training ground for the vast majority of engineers: for those of average ability, and for those who need guidance in subjects not fully investigated in college. These people need clearly written, step-by-step discussions of modern technical subjects which still maintain rigorous standards."

A member from Quebec, Canada, is of the opinion that people working on Antennas may also work on Propagation problems and *vice versa*. They usually have common interests. Because TRANSACTIONS is valuable as a vehicle for having ideas published quickly, some separation from the I.R.E. PROCEEDINGS is desirable. I particularly do not favor "decentralization" of scientific ideas. Consider the plight of the young scientist or engineer who must subscribe to "umpteen" publications in order to keep up to date on information in his field of work. This is a heavy expense.

Kenneth A. Norton of the National Bureau of Standards writes " . . . I think this would be most unwise because recent understanding of antennas and propagation problems have indicated that these two components of the transmission loss are separable (in practice) only in very special cases. This point of view is elaborated in papers by Schott (PROC. I.R.E., June, 1951) and the author (PROC. I.R.E., January, 1953). Both groups of workers should remain conscious of this difficulty. In my opinion, this can best be accomplished by having a single professional group."

Ralph E. Hiatt of Arlington, Mass. writes, "I am not in favor of any further division of the IRE membership. Even now it would be necessary to belong to 5 or 6 groups to keep abreast of developments in my field of interest."

Karl W. Linnes of the Jet Propulsion Laboratory of the California Institute of Technology says, "I oppose splitting the Group in two; I happen to be interested in both Antennas and Propagation. Furthermore, I am interested in the most significant papers in both fields. The excessive number of recent symposia has led to some extremely mediocre papers. Witness those on the West Coast this summer. I think this situation should be avoided in the IRE publications, otherwise they will degenerate to the point of being scientifically worthless, as have those of another engineering society."

On the *pro* side, the Wheeler view has equally strong champions. A Group member from Great Neck, N. Y. writes, "If you are taking a vote on PGAP vs PGA and PGP, I am firmly for the subdivision. The bond between the two subjects is highly artificial. This

is made apparent by the annoyance which papers of one group evoke in readers of the other group."

K. Abbey of San Diego, Calif. feels, "Too much 'propagation' too little 'antennas.' I recognize the arguments against too much specialization and realize that it must stop somewhere, but for what it is worth, I think that 'antennas' should be a separate group."

David Dettinger of Great Neck, N. Y. "Would like to see antennas separated from propagation. The former is my own interest. It's most important in the case of conferences, where I can't just 'not read' propagation articles. It's hard to justify attending a conference concerned mainly with propagation—just to hear a paper or two on antennas."

There is much said—quite eloquently—on both sides. And there is probably still more to say about this subject. Editorially, we will not take up a cudgel for one side or the other. Our aim is to serve as a forum for the membership to express and deliberate their points of view on controversial issues.

Among comments received of general Group interest were those of Carl M. Russell who writes, "In our Antenna and Propagation work at Patuxent River, Md. over the last few years, and in particular during the preparation of 'Ultra High Frequency Propagation' (Wiley), we found many omissions and considerable obsolescence in the existing IRE Standards on Definitions. We attempted to rectify this as best we could, but feel the urgent need for review and publication of a new Standard. How about it? Our group should lead the way in this regard."

Along the same lines, a Cincinnati member opines, "I would like to see Standardization of 'Power Flow/Area.' About 50% of current literature calls this 'Power Intensity' and the other 50% 'Power Density.' It seems to me that 'Power Intensity' more nearly agrees with current usage in the engineering fields of light and sound."

Many excellent suggestions were offered by the membership for improving the TRANSACTIONS. H. W. Lance of Riverside, California writes, "I'd like to see TRANSACTIONS grow into a monthly publication. I especially want to see the summaries of pages (given at various conferences and symposia in the field) continued. One can't attend them all in person, and due to poor verbal presentation one misses the point of those he does hear."

W. Sichak of Nutley, N. J. asks, "Why not add the date on which the paper is received?"

A. J. Simmons of the Naval Research Laboratory, Washington, D. C. wonders about the publication policy of the TRANSACTIONS vs the PROCEEDINGS. "It seems to me that all articles in the area of Antennas and Propagation should first (if worth anything) be printed in the TRANSACTIONS. The editors might select those of greater significance, or of more general interest to pass on to the PROCEEDINGS."

Balth. Van der Pol of Geneva, Switzerland writes, "Would it be possible to internationalize the publication by including contributions from outside the United States?"

G. F. Buranich of Bell Aircraft Corp., Buffalo, N. Y. suggests a question box be included. This would stimulate interest, suggest items and fields of interest, and create more need for publication.

Edward W. Crawford of New York City believes that information about Professional Groups added to the letter to men who are leaving student clubs would bring many new members into the Professional groups, when those students advanced to associate status.

Ed Dyke of Motorola, Inc., Chicago makes a number of interesting comments. "I am very interested in improving the publications technique of professional journals, and from past inquiry have learned that most journals are what they are for economic reasons. From the invitation extended by your postcard, I can mention very briefly a few things that I should hope to see in the future of the publication business, but which may not be practical for your Professional Group publication at present:

a. A question and answer page in which interesting questions would be answered by a specialist to whom the editor directed the question from among those specialists who previously indicated their willingness to be so questioned.
b. Perforations and punched holes to permit removing the most important articles for permanent filing.
c. A modern library-decimal system which might be different for each Professional Group, and with which numbering system every article could be tagged for simplifying the mechanics of filing.
d. A condensation page for each long or major paper, which is part of the paper and contains the most important engineering results graphed and sketched. A less extensive filing system could tear out and collect such condensation pages as a reference, or reminder, which would lead back to library copies of the original paper whenever the fine details of the analysis become important to the user.

As anyone could think of many more such ideas each time he has to look up a paper, I imagine the above list is merely a starting point."

It's unfortunate that we cannot list all comments and suggestions received on both the PGAP and the TRANSACTIONS. They have proved stimulating to the Editors. This correspondence will be circulated among the PGAP Administrative Committee for purposes of information and appropriate action.

This brings us to a point which should be obvious but bears repeating. The vitality of a group depends upon the joined activity of its membership. How productive the PGAP can be in meeting the needs and interests of

individual members depends, of course, on individual participation. Participation need not be limited to concentrated areas where chapters are formed. Geographically isolated members can make their presence felt by means of the Group TRANSACTIONS, whether by contribution of a research paper or by a comment to the *News and Views* section. Let us get together; send us your thoughts about mutual and individual problems.

IRE COMMITTEE ON WAVEGUIDE AND ANTENNAS

In the December, 1953 edition of the PROCEEDINGS, the Antennas and Waveguide Committee of the IRE published a set of standard definitions of waveguide terms.

At the December meeting of the Committee it was realized that the long and arduous task of establishing this set of standards had been completed. The question was then raised: to what real use were these definitions put by people in the industry? It was pretty well recognized by the Committee members that the cost of the effort involved in creating all of these standards was great enough to cause serious speculation on the return from this effort.

Widely varying opinions were offered by the Committee members as to the use of these definitions. Some felt that virtually no use at all was made of these definitions and that the whole effort was largely wasted. The majority, however, believed that these definitions could prove to be very valuable if properly used. A good many subtle points are embodied in the framing of the definitions, and experience in wider use of these terms may prove to be gratifying.

A considerable amount of discussion centered around the IRE method of distribution of the published definitions. It was felt, to some extent, that if the distribution techniques could make it easier for all of the members to procure complete, conveniently collated sets of definitions, the use of these terms might become more widespread.

It might prove very interesting to receive comments from our own professional group members as to their feelings about the use and the dissemination of the definitions; this column would be very receptive to the examination of such comments—reprinting them if sufficient interest is expressed.

GROUP NEWS

Dr. John R. Bolljahn of the Stanford Research Institute has been appointed the PGAP Administrative Committee representative of the West Coast Convention Technical Program Committee for 1954. Dr. Bolljahn, as Chairman of the Meetings Committee, is also organizing the Symposium for the 1954 IRE National Convention, which was voted at the last August meeting of the Administrative Committee. The subject to be chosen seems likely to be UHF vs VHF propagation as it affects recent TV broadcasting experience. Delmer Ports has suggested the session be listed as follows:

Antennas and Propagation Symposium
UHF TV—Boom or Bust?

At a recent meeting of the PGAP Administrative Committee, some thoughts on the programming of this Symposium were expressed as follows: The Chairman to be chosen will be one who has no connection with any commercial organization directly interested in the TV broadcasting field. There will be a minimum of three speakers. The first speaker is contemplated to reflect the FCC attitude, and no doubt connected with the agency. One speaker should represent the Bureau of Standards. A third speaker might be one who could present the results obtained in practice as compared to theoretical predictions. A fourth man might represent one of the TV broadcasting companies. One of the speakers should include in his talk the role which antennas have had to play in the experimental results obtained. This Symposium should turn out to be a stimulating affair like one or two of our sessions of previous years when the meeting room was 200% filled to capacity. These sessions could have continued a week, if time had permitted.

The PGAP TRANSACTIONS is becoming available for subscription to libraries, and other interested organizations and individuals.

It has been suggested, at a meeting of Professional Groups Committee, that since a number of excellent papers may have to be turned down for presentation at the National Convention, authors should be advised to submit these papers for publication in the TRANSACTIONS of their Group. As far as the PGAP goes, Chairman P. S. Carter has proposed that all authors, whether or not their papers are accepted for presentation at the Convention, submit their papers to the Editor of ANTENNAS AND PROPAGATION. Those papers judged by the Review Committee to be of general interest to the majority of IRE members, will be recommended for publication in the PROCEEDINGS.

Martin Katzin has been appointed as the Propagation Committee representative to the Antenna Committee.

GROUP CHAPTERS AND CHAPTER NEWS

We are pleased to welcome the new PGAP Chapter in the Albuquerque Section.

PGAP members in the San Diego area—please take note! Dr. John B. Smyth of the U. S. Navy Electronics Laboratory is attempting to form a chapter. Your interest and assistance are requested.

Delmer Ports wants to know what is wrong with the Washington area. There are approximately eighty professional group members and no activity toward forming a local chapter. While unable to devote full energy

to the task himself, Delmer will be glad to help the chapter get started.

William D. Neal of Neal Electronic Co., Huntsville, Ala., presented a talk on the Design of TV Antennas at the August meeting of the Huntsville subsection of the IRE.

Chapter News

The Los Angeles Chapter of the PGAP has increased its membership from 50 to approximately 140. The following chapter meetings and talks have been held during the past months:

June 9—Dr. J. Smyth, Head, Atmospheric Studies, Naval Electronics Laboratory, San Diego, spoke on "Propagation in the Troposphere." Calculated values of the propagation constants of the present theories were compared to measurements. The attendance was 42.

August 18—Mr. F. Zucker of Air Force Cambridge Research Center spoke on "Surface Waves." The paper covered theories of single surface transmission lines and a comparison of theoretical and experimental values of propagation constants; local field distributions, and radiation patterns were made. The attendance numbered 35.

Plans are under way for a joint meeting of the Los Angeles Chapter with the Airborne Electronics Chapter and a group of aerodynamicists from local aircraft companies. The meeting will be an open discussion related to the problems of joint attack on the problem of integration of the electronics, antennas, and the airframe in the earliest preliminary design stages rather than a belated afterthought.

URSI Fall Meeting 1953

The U.S.A. and Canadian National Committee of the International Scientific Radio Union together with the Professional Group on Antennas and Propagation of the Institute of Radio Engineers sponsored this first joint U.S.A.-Canadian technical meeting of URSI. Approximately 200 radio scientists attended.

The hosts were the Radio and Electrical Engineering Division of the National Research Council of Canada, and the Radio Physics Laboratory of the Canadian Defence Research Board.

Fifty-six papers were presented covering the fields of commissions:

Commission 1. Radio Measurement Methods and Standards.
Commission 3. Ionospheric Radio Propagation.
Commission 4. Terrestrial Radio Noise.
Commission 5. Radio Astronomy.

The program, containing abstracts of the papers, is available from J.C.M. Scott, Secretary, Canadian National Committee URSI Defence Research Board, Ottawa, Canada.

Preparations and planning for the XIth General Assembly of URSI to be held in The Netherlands from August 23 through September 2, 1954 occupied the U.S.A. National Committee at their administrative meeting. The URSI Spring Meeting, 1954 is scheduled for Washington D. C. on May 3, 4, 5, and 6.

IRE National Convention

The 1954 IRE National Convention, to be held on March 22–25 at the Waldorf-Astoria Hotel and Kingsbridge Armory in New York City, promises to be the largest and most important technical meeting of the year. The Radio Engineering Show with its 600 engineering exhibits will be held in a new location, the spacious Kingsbridge Armory in the Bronx. Fifty-one technical sessions are scheduled at the Armory, the Waldorf-Astoria, and the nearby Hotel Shelton. Special IRE busses, free of charge to registrants, will run at frequent intervals between the Waldorf-Astoria and the Kingsbridge Armory. In addition, both locations are within one block of subways.

In addition to the symposium mentioned previously, the PGAP has organized three general sessions on antennas and propagation. All four sessions will be held at the Kingsbridge Armory, starting Wednesday morning, March 24, and running through Thursday. Listed below is the program of the PGAP sessions.

Antennas and Propagation I—General

John T. Bolljahn, *Chairman*
Stanford Research Institute

"Empirical Approximations to the Current Values for Large Dolph-Tchebyscheff Arrays," Louis L. Bailin, Robert S. Wehner, and Ivan P. Kaminow, Research and Development Laboratories, Hughes Aircraft Company

"Gain Pattern of a Terminated-Waveguide Slot Antenna by an Equivalent Circuit Method," Leopold B. Felsen, Microwave Research Institute, Polytechnic Institute of Brooklyn

"A Four-Slot Cylindrical Antenna for VOR Service," R. M. Sprague and Andrew Alford, Andrew Alford, Consulting Engineers

"Trapped Wave Antennas," H. Ehrenspeck, W. Gerbes and F. J. Zucker, Air Force Cambridge Research Center

"Scattering of Electromagnetic Waves by Wires and Plates," Joseph Weber, Naval Ordnance Laboratory

Antennas and Propagation II—Microwave Antennas

Garnet A. Woonton, *Chairman*
Eaton Electronics Research Laboratory, McGill Univ.

"Reflections in Microwave Antennas and Their Harmful Effects," Peter W. Hannan, Wheeler Laboratories, Inc.

"Surface Matching of Dielectric Lenses," E. M. T. Jones and S. B. Cohn, Stanford Research Institute

"Double Parabolic Cylinder Pencil Beam Antenna," Roy C. Spencer, F. Sheppard Holt, Helen Beauchemin, and John Samson, Air Force Cambridge Research Center

"Diffuse Radiation in Pencil Beam Antennas," David Carter, Consolidated Vultee Aircraft Corporation

"Theoretical Gain of Flat Microwave Reflectors," D. R. Crosby, Communications Engineering Section, RCA Victor Division

Antennas and Propagation III

Henry G. Booker, *Chairman*
Cornell University

"Isotropic Variable Index Media," W. O. Puro and K. S. Kelleher, Melpar, Inc.

"The Characteristics of a Vertical Antenna with a Radial Conductor Ground System," James R. Wait and W. A. Pope, Radio Physics Laboratory, Defense Research Board

"Toward an Information Theory of Propagation Through Time Varying Media," J. Feinstein, National Bureau of Standards

"Comparative 100 MC Measurements at Distances Far Beyond the Radio Horizon," Albrecht P. Barsis, National Bureau of Standards, Tropospheric Propagation Research Section

"The Measurement of the Polarization of Radio Waves Reflected from the Ionosphere at Non-Vertical Incidence," George T. Inouye, Technicolor Motion Pictures Corporation

Antennas and Propagation IV—Symposium: UHF Television—Boom or Bust

John B. Smyth, *Chairman*
Atmospheric Studies Branch,
U.S. Navy Electronics Lab.

"FCC Rules and Propagation Data," E. W. Allen, Federal Communications Commission

"Propagation in the UHF TV Band," J. W. Herbstreit, National Bureau of Standards, Propagation Research

"Overcoming the Line of Sight Shibboleth with the Air and High Power," T. J. Carroll, Massachusetts Institute of Technology, Project Lincoln

"A Comparison of the Antenna Problems in UHF and VHF TV," Lloyd Krause, Commercial Equipment Electronic Division, General Electric Company

contributions

UHF Omnidirectional Antenna Systems for Large Aircraft*

W. SICHAK† AND J. J. NAIL‡

Summary—This paper discusses the problem of obtaining omnidirectional coverage from antennas operating between 1,000 and 3,000 megacycles on large aircraft. Electromagnetic modeling was used to determine the limitations of several single antenna sites on typical commercial aircraft. Considering all azimuth angles and ±30 in elevation to be equally important, the best coverage obtainable from a single radiator is equivalent to the radiation from a free-space dipole for 50 per cent of the time. To improve this, dual antenna systems must be used.

Dual antenna requirements depend on whether or not the airborne equipments know when they should be receiving a signal. The distance-measuring equipment (DME) is a typical system that knows when it should be receiving a signal, while radar safety beacon equipment does not know when or from what direction it is being interrogated. Direct parallel feed, the least complicated method of operating dual antennas, allows simple hybrid multiplexing to be used. With this type of operation interference occurs where the individual patterns overlap. Performance in this region is investigated on a probability basis for beacon operation and found favorable; for DME this region is uncertain.

In addition, performance is predicted when the RF voltage in one of the dual antennas is (a) shifted periodically in phase, (b) delayed, and (c) interrupted periodically. Considerations involved in an antenna system common to DME and beacon are discussed.

* This paper covers a portion of the work being done under Bureau of Aeronautics contract NOa(s)-12212 for the Air Navigation Development Board. This project is one of a group initiated to provide basic information for the development of a common military /civil system of aids to air navigation. The Air Navigation Development Board (ANDB) was established by the Departments of Defense and Commerce in 1948 to carry out a unified development program aimed at meeting the stated operational requirements of the common military/civil air navigation and traffic-control system. This project, sponsored and financed by the ANDB, is a part of that program. The ANDB is located within the administrative framework of the Civil Aeronautics Administration for housekeeping purposes only. Persons desiring to communicate with ANDB should address the Executive Secretary, Air Navigation Development Board, Civil Aeronautics Administration, W-9, Washington 25, D. C.

† Federal Telecommunication Labs., a division of the International Telephone and Telegraph Corp., Nutley, N. J.

‡ Formerly with Federal Telecommunication Labs., now with Bell Telephone Labs., Whippany, N. J.

Introduction

Above 1,000 mc the airborne antenna problem becomes easier in some respects, but obtaining omnidirectional coverage (the usual requirement) becomes considerably more complicated than at lower frequencies. It does not appear possible to obtain omnidirectional coverage with a single antenna on large aircraft, so that systems requiring this coverage must use two or more antennas.

It is reasonable to assume that individual equipment requirements will vary considerably; consequently this problem had to be approached from the viewpoint of supplying information that allows a choice to be made between performance and complexity, weight, and expense. The aim of this paper is to present information on single antennas as well as dual antenna systems so that a reasonable choice can be made to fit a specific system.

Method of Measurement

Modeling appeared to be the most practicable method if the Fraunhofer condition (phase error across the largest dimension less than about 1/16th wavelength) could be violated. It appeared that this could be done since the radiator and its immediate surroundings could be properly illuminated while reflections from the extremities of the model would cause, primarily, errors in locating the true position of the minima or maxima.

Fig. 1(a) shows a pattern (θ polarization) of a quarter-wave monopole on a 1/24th-scale model of a DC-3 (94-wavelength wingspan) with a transmitter-model separation of 2.1×10^4 wavelengths. Fig. 1(b) shows the patterns with 1.2×10^3 wavelength separation. The calculated maximum phase errors across the wingspan are 0.056 (20 degrees) and 1.0 (360 degrees), respec-

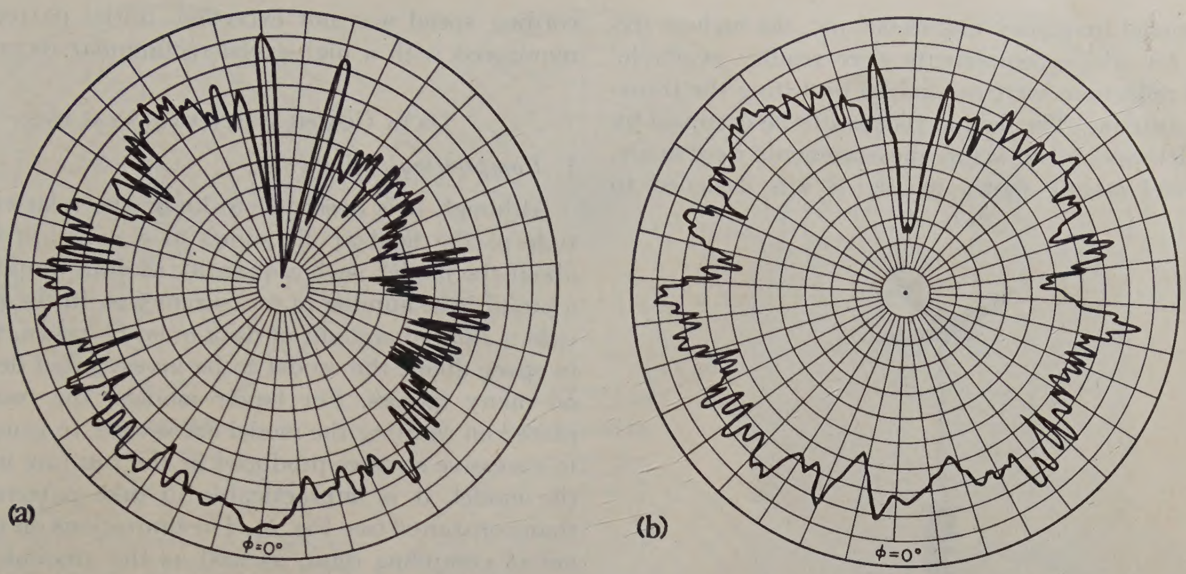


Fig. 1—Effect of separation between transmitter and aircraft model on measured power patterns. (a) is for a separation of 2.1×10^4 wavelengths and a 0.056-wavelength maximum phase error across the wingspan. (b) is for 1.2×10^3 wavelengths separation and a 1.0-wavelength maximum phase error across the wingspan.

tively. The differences in the patterns are minor. (Note that these two patterns are plotted in power; all other patterns herein are plotted in voltage.) Small transmitter-model separations can introduce serious errors in measuring the characteristics of certain types of direction-finding antennas and broadside and end-fire arrays of large aperture, so that small transmitter-model separation should be used with caution.

Except for the distance between antennas and the height and strength of the model mount, the modeling system is the conventional one. (See Fig. 2.) Fig. 3 shows range with rotating mount.

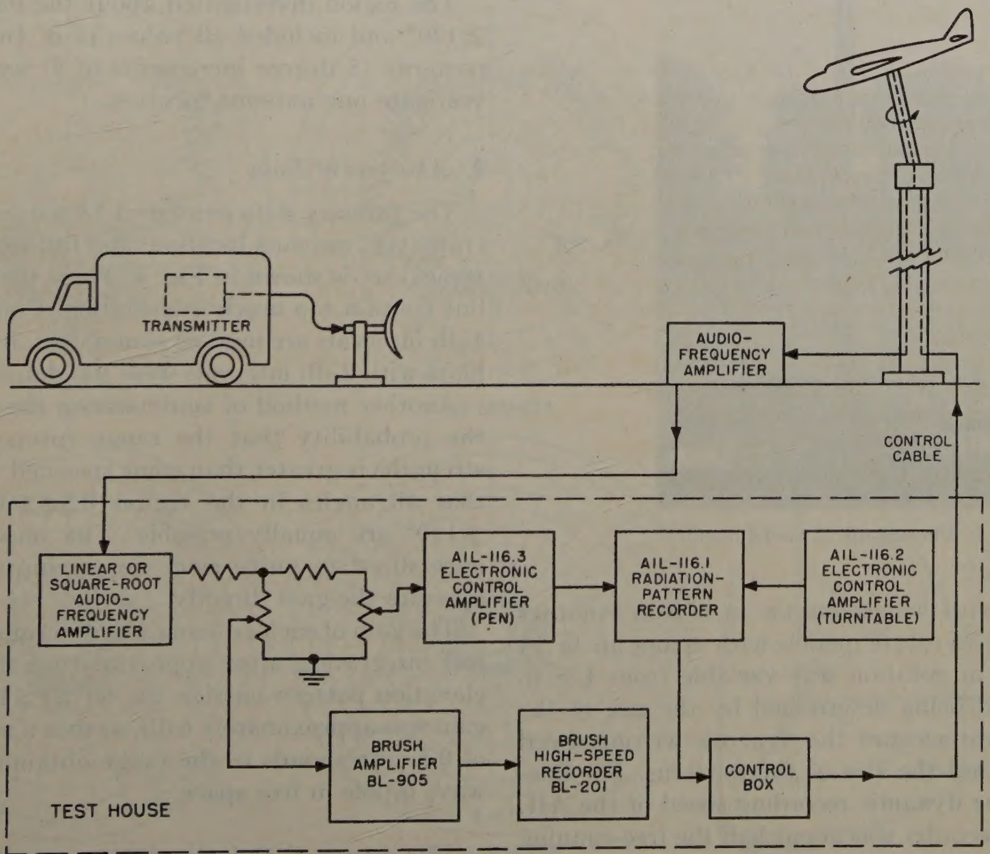


Fig. 2—Block diagram of model range. The 24,000-mc transmitter was mounted in a truck and the 2-foot paraboloidal antenna placed outside of the truck. The distance to the model was varied up to 900 feet. The crystal detector from which the receiver obtained its signal was inside the aircraft model. The output from the audio-frequency amplifier at the base of the model mount passed through a shielded cable for about 150 feet to the test house.

The model frequency was 24,000 mc, the highest frequency for which components were readily available. Ground reflections were minimized by tilting the transmitting antenna. The field in the area to be occupied by the model was probed before measurements were made. The model mount shown in Fig. 3 was designed to handle models with wingspans up to 6 feet. Another mount was built to rotate models with spans up to 14 feet. The speed of rotation was variable from 1/5 to 1 rpm, the speed being determined by the size of the model, taking into account the dynamic writing speed of the recorder and the size of the patterns. For fine-lobe structure the dynamic recording speed of the AIL Type 116 polar recorder was about half the free-running speed of 15 inches/second. To insure that allowable recording speed was not exceeded, initial patterns were monitored with a high-speed rectangular recorder.

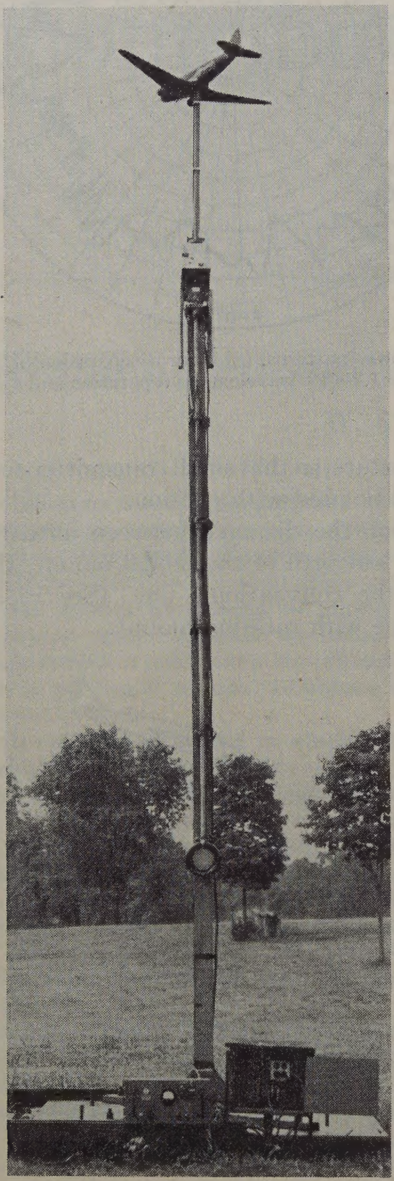


Fig. 3—Photograph of model mount.

Data Compilation and Analysis

1. *Compilation of Data*

Although it is desirable to know the relative amplitude of the field at all points at a constant distance about the model, such a quantity of data would require a prohibitive number of measurements. In the practical case, a limited amount of data must be taken, the path in space about the model to be investigated depending on many factors. For larger models, the restrictions placed on rotating the model are severe. In general, due to excessive stresses produced in the rotating mount or the model, it is impracticable to take patterns other than constant θ (see Fig. 4). The limitations on the manner of compiling data, as well as the amount of data required, are both directly proportional to plane size and frequency. However, there is some compensation in the fact that the larger planes do not require coverage at all values of θ.

Obviously errors may be encountered in interpolating between data taken along constant-θ paths if the increment in θ is not made much smaller than the minimum lobe width in the interference pattern. Before deciding what increment of θ to use, the model may be nodded and the vertical pattern noted at strategic values of ϕ.

The region investigated about the model was $60° \geqq \theta \geqq 120°$ and included all values of ϕ. In most cases, 13 patterns (5 degree increments of θ) were taken to investigate one antenna location.

2. *Analysis of Data*

The primary data consist of 13 patterns for each aircraft type, antenna location, and full-scale frequency. A typical set is shown in Fig. 4. These patterns are useful but contain too much information. Contour plots with 4-db intervals are used as summaries; it was found that plots with 2-db intervals were too detailed.

Another method of summarizing the data is to show the probability that the range (proportional to field strength) is greater than some specified value, assuming that all angles in the region $0 \leqq \phi \leqq 360°$ and $60° \leqq \theta \leqq 120°$ are equally possible. The maximum range is normalized to unity since no attempts were made to measure the gain directly.

The gain of each antenna was determined[1] by mechanical integration after approximating the shape of the elevation pattern outside the $60° \leqq \theta \leqq 120°$ region. The gain was approximately 6 db, so that a normalized range of 0.5 corresponds to the range obtainable with a half-wave dipole in free space.

[1] For example, E. C. Jordon, "Electromagnetic Waves and Radiating Systems," Prentice-Hall, Inc., New York, N. Y., p. 420; 1951.

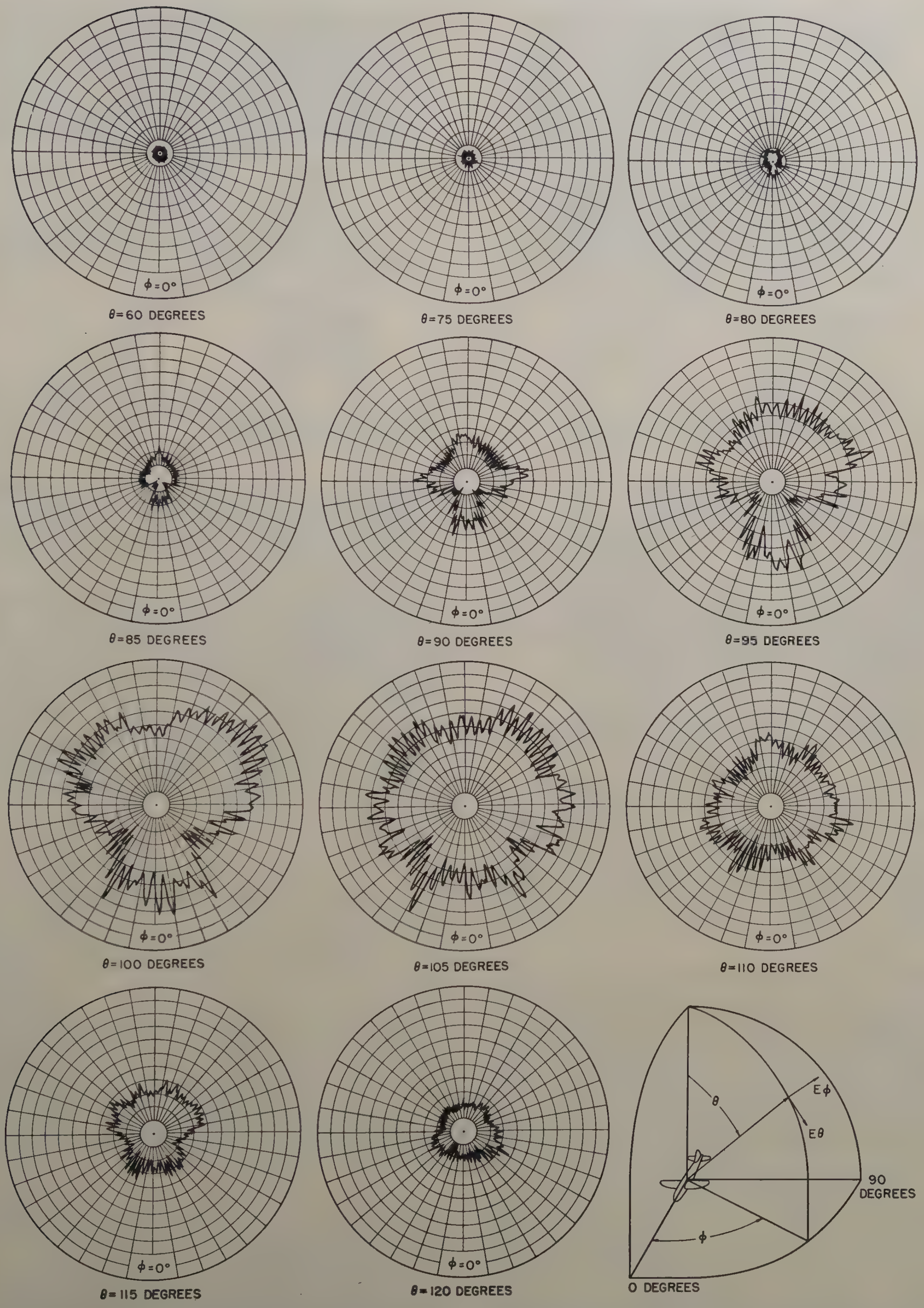


Fig. 4—Typical set of field-intensity patterns for a quarter-wavelength stub antenna mounted on the bottom of the fuselage at the center of the wing root of a 1/24-scale model of a DC-3 aircraft. Vertically polarized signals transmitted over a 900-foot path are plotted in voltage for all values of ϕ, with zero corresponding to the nose of the aircraft. Diagrams for $\theta = 65$ and 70 degrees have been omitted as not differing significantly from the adjacent plots.

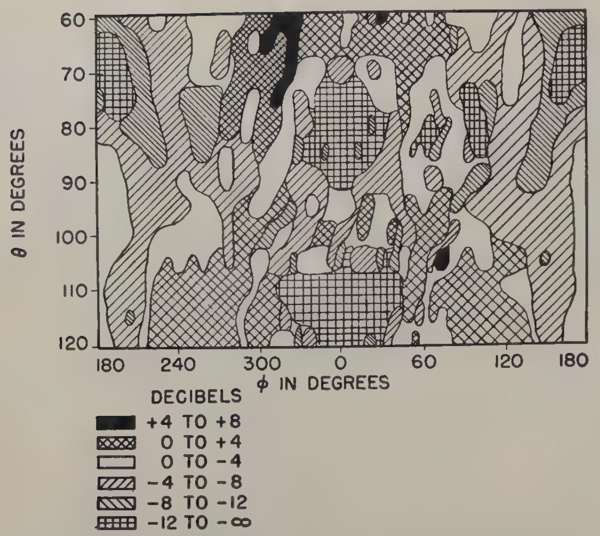


Fig. 5—Contour map for a discone antenna mounted on the vertical stabilizer of a DC-3 at 1,000 mc.

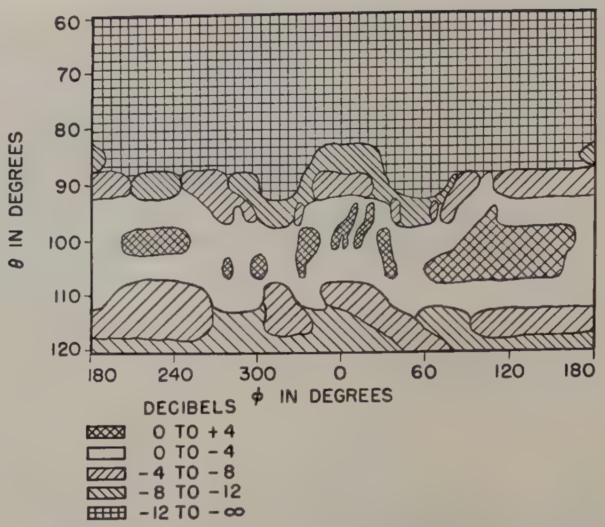


Fig. 6—Contour map for a quarter-wavelength stub antenna mounted on the bottom of the fuselage on the center line of the wings of a DC-3 at 1,000 mc.

Single Antennas

The radiation patterns of single antennas at various locations on several representative models were studied. Contour maps showing typical results appear in Figs. 5 through 11 and a probability curve is shown in Fig. 12. Dots on some contour maps indicate sharp lobes (less than 5 degrees wide) at least 2 db above the background. Table I summarizes the results obtained.

Table I

PROBABILITY OF OBTAINING FREE-SPACE-DIPOLE-RANGE (ALL POINTS IN AZIMUTH AND ±30° IN ELEVATION OF EQUAL IMPORTANCE)

Aircraft Type	Top of Fuselage	Bottom of Fuselage	Top of Vertical Stabilizer	Frequency mc/Sec
DC-3	50	27	21	1,000
XB-51	14	22	22	1,200
DC-6	32	46	13	1,500
DC-3	47	42	—	3,000

The site atop the vertical stabilizer was uniformly poor as a result of the finer-lobe structure in the vertical interference pattern. The coverage on the XB-51 could be improved by selecting a site to minimize the azimuth angle over which strong wing interference is encountered. All top and bottom fuselage sites were chosen opposite the wing-root, the optimum position for aircraft with straight wings. For aircraft with swept-back wings the optimum site for fuselage antennas is probably forward of the wing root. The coverage from the bottom fuselage site on the DC-3 at 1,000 mc suffers from shadowing and reflections from the engine nacelles. This same site gave better coverage at 3,000 mc. The reason for this is not clearly understood. However, this particular site places the antenna over the equivalent of an irregularly shaped ground screen so that the effect of varying frequency is difficult to predict. The top fuselage site on the DC-6 produced a coverage figure somewhat below that obtained from the DC-3. This is caused by the flat-top characteristic of the DC-6 fuselage which caused the resultant vertical pattern to tilt up about 20 degrees.

In general, the performance of aircraft antennas on large aircraft is more easily predicted[2] at UHF than at VHF and below. As might be expected specular reflection was encountered and, within practical limits, scattering could be predicted from scalar field theory.[3] Although in some locations improvement in coverage may be obtained by shaping the free-space pattern of the radiator so as to minimize reflections, there is nothing that can be done in the way of antenna design to reduce shadowing.

Dual Antenna Systems

Air-navigation and traffic-control equipments can be divided into two classes: (*a*) that class which knows when it should be receiving a signal, such as the distance-measuring equipment (DME)[4] and (*b*) that class which does not know when it should be receiving a signal, such as the radar safety beacon.[5] For systems of the former class, the space about the aircraft may be divided into sectors with only one sector covered at any instant of time, while systems of the latter class require coverage that is equally receptive over some finite time interval to signals from any point in azimuth and ±30 degrees, approximately, in elevation.

[2] R. H. J. Carey, "A survey of external and suppressed aircraft aerials for use in the high frequency band," *Proc. IEE* part III, vol. 99, pp. 197–209; July, 1952.

[3] H. J. Riblet and C. B. Barker, "A general divergence formula," *Jour. Appl. Phys.*, vol. 19, pp. 63–70; January, 1948.

[4] R. C. Borden, C. C. Trout, and E. C. Williams, "Description and evaluation of 100 channel distance measuring equipment," Proc. I.R.E., vol. 39, pp. 612–618; June, 1951.

[5] G. Heath, "Air traffic control system," *Electronics*, vol. 25, pp. 152–154, June, 1952.

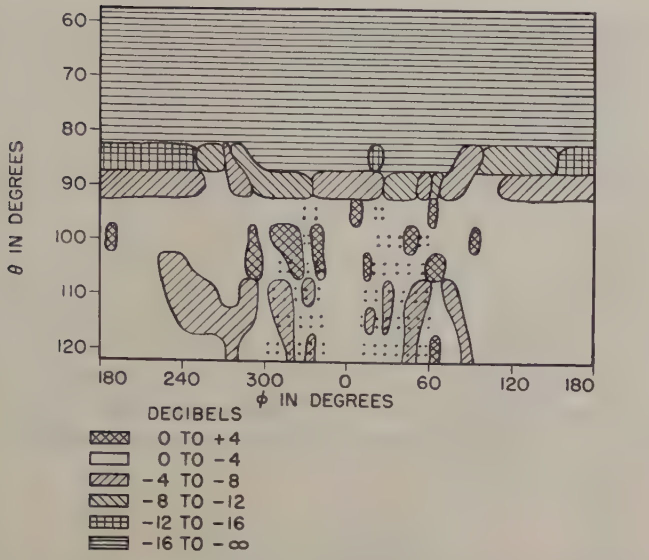


Fig. 7—Contour map for a quarter-wavelength stub antenna mounted on the bottom of the fuselage on the center line of the wings of a DC-6 at 1,500 mc.

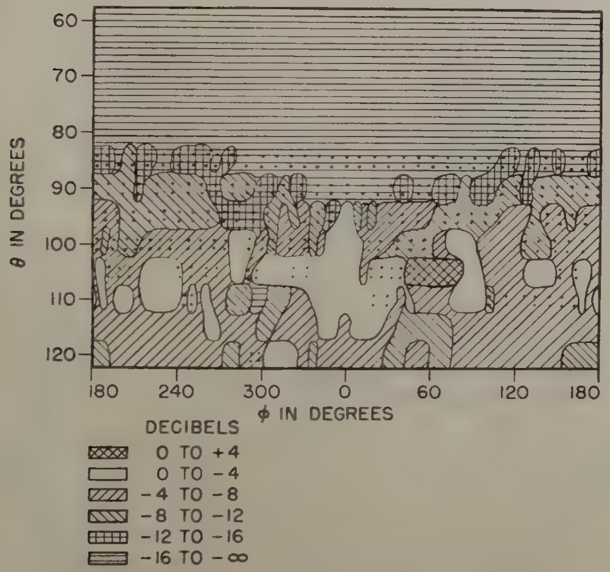


Fig. 8—Contour map for a quarter-wavelength stub antenna mounted on the bottom of the fuselage on the center line of the wings of an XB-51 at 1,200 mc.

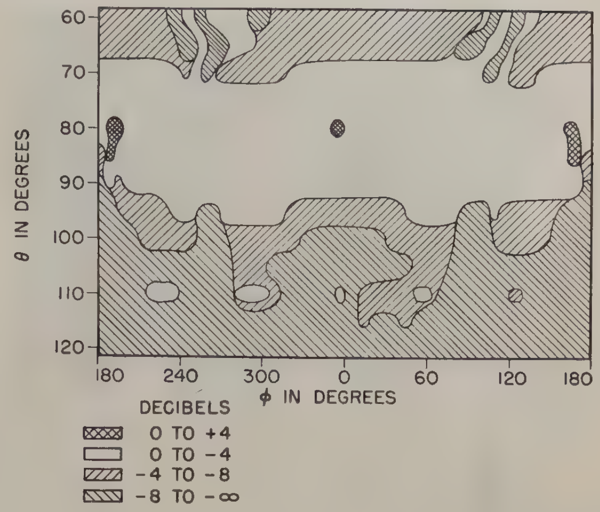


Fig. 9—Contour map for a quarter-wavelength stub antenna mounted on the top of the fuselage on the center line of the wings of a DC-3 at 1,000 mc.

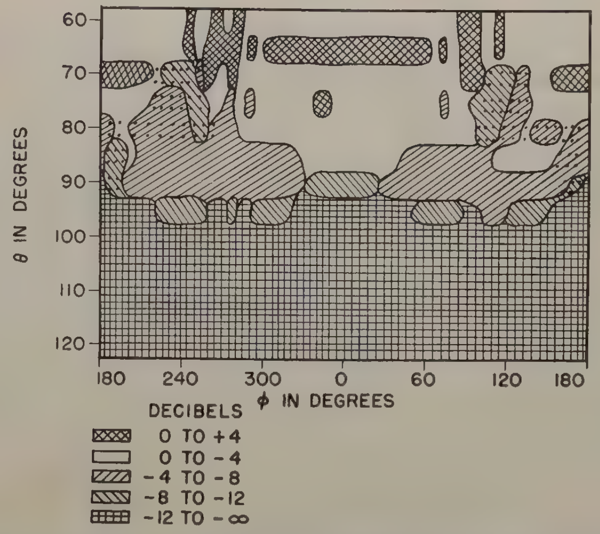


Fig. 10—Contour map for a quarter-wavelength stub antenna mounted on top of the fuselage near the center line of the wings of a DC-6 at 1,500 mc.

In considering means of utilizing dual antennas the following outline will be followed:

A. Passive systems: (1) dual antennas in parallel, and (2) dual antennas with a delay line.

B. Systems using moving parts: (1) dual antennas with an RF switch, (2) dual antennas with a phase shifter, and (3) common antenna system.

A. *Passive Systems*

A-(1). *Passive Systems: Dual Antennas in Parallel.* The least-complicated system is to operate in parallel dual antennas with complementary patterns. Interference is encountered where individual patterns overlap (see Fig. 13). Fundamental disadvantages are (1) long cables are sometimes required; (2) in all cases a 3-db junction loss must be considered.

It will be shown that the radar safety beacon can operate satisfactorily in certain types of interference regions. The performance of the present DME in an interference region is uncertain. It appears that some portions of the interference regions may be usable, but for a conservative analysis the interference regions will be considered unusable for DME.

DME. Wingtip antennas, each antenna having essentially a cardioid pattern, is one possible system; nose and tail antennas of similar type is another system. Both systems have interference only at certain values of ϕ but require long transmission lines. Antennas located top and bottom of the fuselage produce vertical interference making DME operation uncertain at all values of ϕ and hence will not be considered.

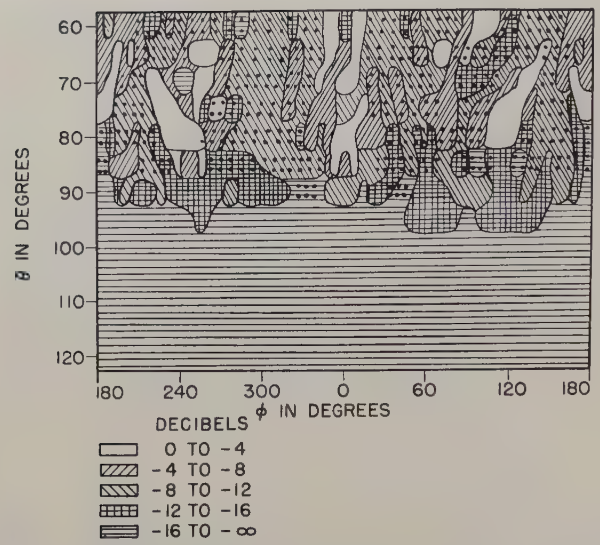
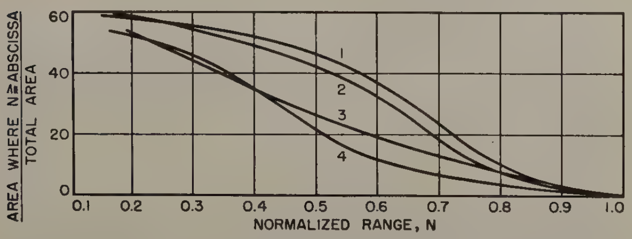


Fig. 11—Contour map for a quarter-wavelength stub antenna mounted on top of the fuselage on the center line of the wings of a XB-51 at 1,200 mc.

An antenna with a cardioid pattern in the horizontal (ϕ) plane and enough vertical (θ) directivity to give a maximum average gain of 5 db over a dipole is about the highest-gain antenna that can be used if vertical coverage between $\theta=60$ degrees and 120 degrees is required. Taking transmission-line loss and junction loss into account, the maximum range that can be expected is free-space-dipole range. If a range 70 per cent of free-space-dipole range is usable, acceptable performance can be obtained 45 per cent of the time. In the event a smaller sector, say ± 15 degrees in elevation, can be used, free-space-dipole range can be obtained 45 per cent of the time.



Curve	*Airplane*	*Scale*	*Frequency in Megacycles*
1	DC-6	1/16	1,500
2	DC-3	1/8	1,000
3	DC-3	1/24	3,000
4	XB-51	1/20	1,200

Fig. 12—Probability that the range exceeds a specified value for a quarter-wavelength stub antenna mounted on the bottom of the fuselage of the airplane.

Beacon. For simplicity, assume that cardioid antennas are mounted on the nose and tail. The coverage is good in two symmetrical sectors fore and aft, but toward the sides there will be narrow lobes and the coverage there may or may not be good, depending on many factors. If the interrogating radar is never more than a few degrees off the course of the airplane, the coverage will be good. If, on the other hand, the interrogating radar is in the interference region off the wing, the coverage may become uncertain.

The geometry is shown in Fig. 13. Near $\phi=90$ degrees the minima approach zero making this region the most critical one. As the aircraft flies at a constant velocity V, the rate of change of angle with respect to a ground radar (located at $\phi=90$ degrees and a minimum distance d_0 between line of flight and ground station) is $(d\phi/dt_0) = V/d_0$. Plotted in Fig. 14 is the normalized rate of change of ϕ as a function of distance along the line of flight. The time required for one lobe to move past the ground radar is

$$T = \frac{3600}{V_m}\left(\frac{\lambda}{l}\right)\frac{d_0}{\sin^3\phi}$$

where

V_m = velocity in miles per hour,
d_0 = minimum distance between line of flight and ground station in miles,
l = separation between antennas, and
λ = wavelength.

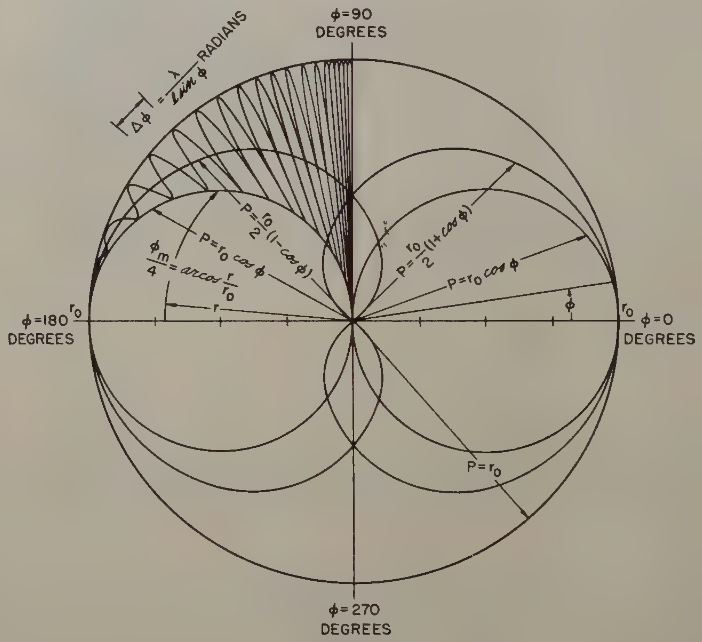


Fig. 13—Resultant of two cardioid patterns connected in parallel. r_0 = maximum range, l = spacing between antennas, ϕ_m = total angle for which the range $\geq r$.

Thus the time required for one lobe to move past the ground radar varies from 6.3 to 18 seconds as ϕ varies from 90 to 45 degrees (velocity = 200 mph, l = 100 feet, frequency = 2,700 mc, d_0 = 100 miles).

Fig. 15 shows how much of each lobe in the interference region exceeds a certain amplitude. This figure gives the probability of receiving a single interrogation.

This probability is correct only for the case where the aircraft is in the beam of the interrogating radar for a time that is considerably less than the time required for one lobe to move past the interrogating radar. This assumption is not valid when the ground antenna is searchlighting the airplane.

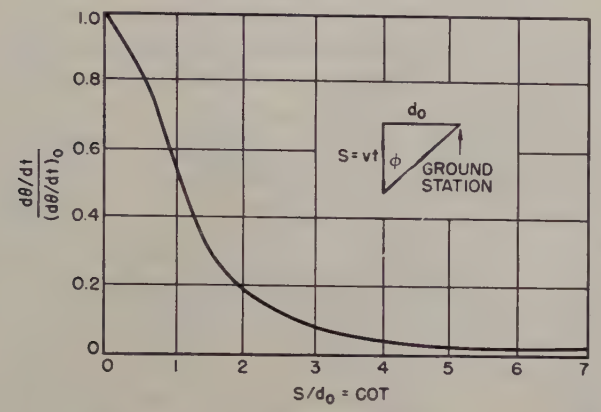


Fig. 14—Normalized rate of change of an angle $d\phi$ versus distance $S/d_0=\cot\phi$. $(d\phi/dt)/(d\phi/dt)_0=1/[1+(S/d_0)^2]$.

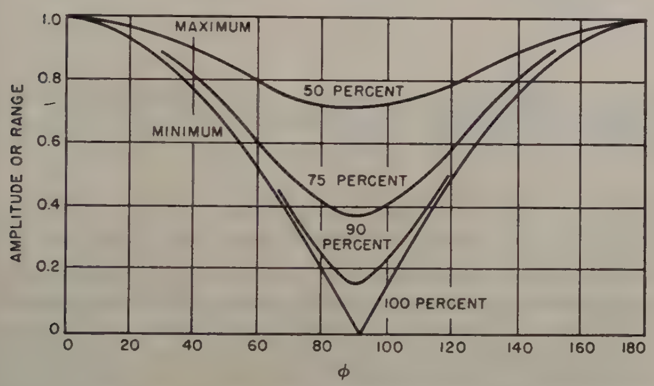


Fig. 15—Percentage of each lobe in the interference region exceeding a certain amplitude plotted against the angle ϕ.

To a first approximation, the motion of the airplane is essentially smooth and the ground-radar antenna rotates at a regular rate so that the problem becomes one of periodic sampling of a periodic wave. If the airplane transmitted continuously, the field strength at the ground radar would be periodic in time, as shown in Fig. 16(a). (The period is not constant, but for small time intervals it is approximately so.) As the radar antenna scans, it samples the periodic wave. If the period of the wave and sampling interval are known, a diagram similar to Fig. 16(b) can be drawn in which the length of a segment represents the time during which the signal is above the threshold and the gap represents the time during which the signal is below the threshold. The probability of one hit in one try is the same as that given above. The probability of two hits out of two tries is equal to $P_1P(\tau)$, where P_1 is the probability of one hit in one try at time T and $P(\tau)$ is the probability of getting a hit on the second try at time $T+\tau$ if the first try were a hit. $P(\tau)$ has the form shown on Fig. 16. Diagrams are shown in Fig. 17. To construct these curves draw a straight line from $P=1$, $\tau=0$ to $P=0$, $\tau=H$; draw a straight line from $P=0$, $\tau=H$ to $P=1$, $\tau=H+M$. If these lines overlap, add the P's in the overlap region.

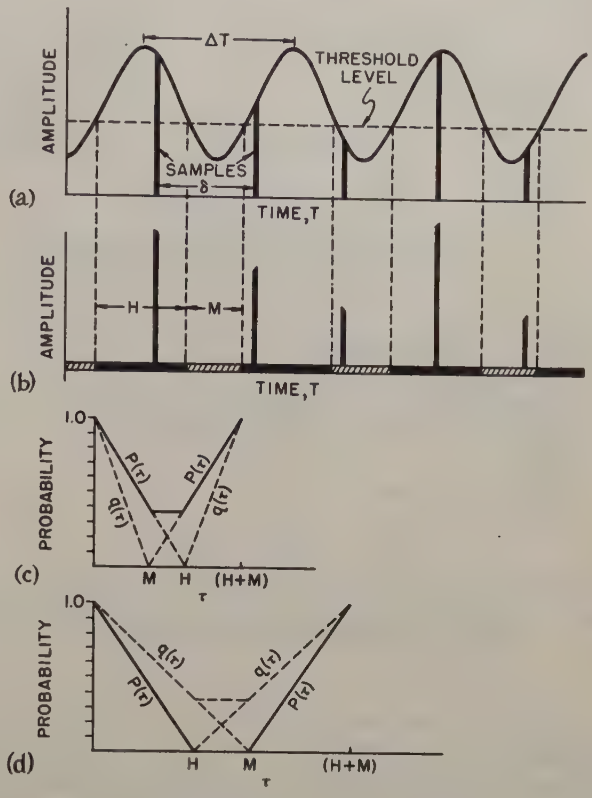


Fig. 16—Periodic signal being sampled at time intervals δ is shown at a; the sampling diagram is at b. The solid base line in b indicates the signal to be above the threshold level and the shaded line indicates it to be below threshold value. c and d are probability diagrams for $M>H$, respectively.

The probabilities for various combinations are as follows:

1	*2*	*Probability*
Hit	Hit	$P_1P(\tau)$
Miss	Miss	$(1-P_1)q(\tau)$
At least one hit		$1-(1-P_1)q(\tau)$

If the first try at time T were a miss, $q(\tau)$ is the probability of getting a miss at time $T+\tau$.

The minimum value of P_1 and $P(\tau)$ for any set of conditions will be taken since no assumptions will be made concerning the sampling rate; conversely the maximum values of q_1 and $q(\tau)$ will be taken. This means that the expression for at least one hit in two tries reduces to P_1 when $q(\tau)=1$.

The probabilities considered here are independent of the rate at which the lobes in the interference region move past the ground station, provided the time required for one lobe to move past the radar is less than the time

per scan of the ground antenna. For nose and tail or wingtip antennas on large aircraft there is no advantage in high-speed switching or phase shifting.

Following this procedure and using the same constants used in the previous example, the probability plots shown in Figs. 17 and 18 were made.

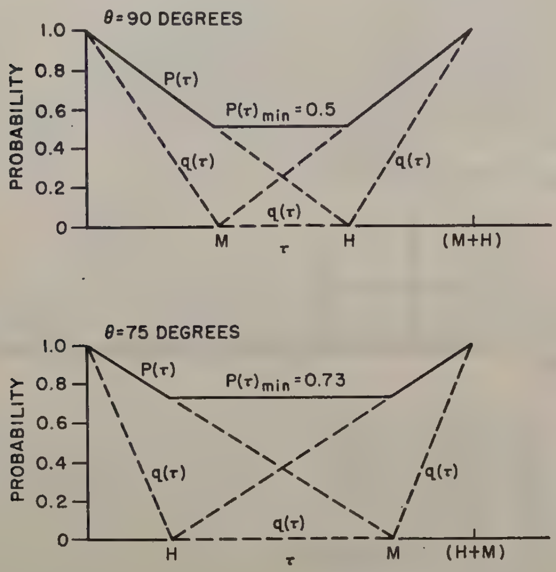


Fig. 17—Probability diagram applicable to a DC-6 at 2,700 mc when $V_m=200$, $l=100$ feet, and $d_0=100$ miles.

A-(2) *Dual Antennas Connected through a Delay Line (for Beacon).* Locate two antennas with complementary coverage on the aircraft (Fig. 19(a)). Now connect the two antennas together but place in one antenna lead a delay line with a time delay of the order of one pulse

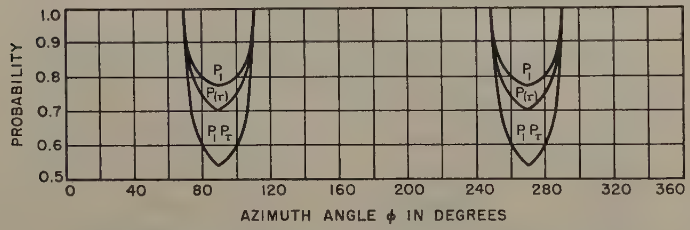


Fig. 18—Probability versus azimuth angle for the same parameters of Fig. 17. The usable range is 0.35 of the maximum range.

length and connect the combination to the transmitter-receiver (Fig. 19(b)). When an interrogating pulse is received, the output of antenna A cannot interfere with the output of antenna B because output B is delayed by a whole pulse length. The receiver input, therefore, will be one of the forms shown in Fig. 19(c). When the transmitter replies, the input to the ground receiver is also, by reciprocity, one of the forms shown in Fig. 19(c).

The delay line can be placed in the ground equipment if orthogonal polarizations are used. Two ground antennas are needed, with the delay line connected to one antenna only. All of the antennas must be relatively free of cross polarization at angles at which both receive to prevent interference.

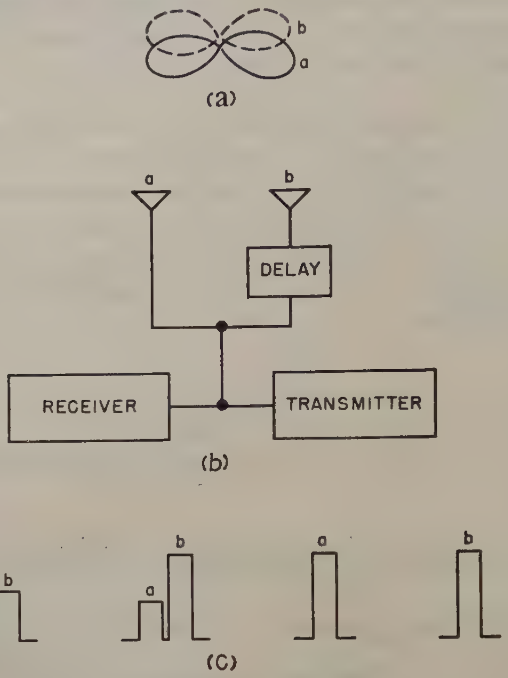


Fig. 19—Dual antenna with delay line: *a* is the pattern overlap, *b* is the arrangement of the equipment, and *c* is the input waveforms to the receiver.

A rotary phase shifter can be used instead of the delay line. Its advantages are smaller size and less loss. Its disadvantages are that some pulses may be lost and that it is a rotating device which needs power and maintenance. For ground use, the delay line may be preferable. The major disadvantage is loss in the delay line. The advantage is that all circuits are passive. If phase shifters are used the major disadvantages are complexity, size, and weight; the advantage is less insertion loss.

B. *Systems Using Moving Parts*

B-(1). *Dual Antennas Connected Through an RF Switch (for DME).* This is an obvious solution for DME since it knows when it should be getting a signal. For the case of single $\lambda/4$ stub antennas located top and bottom of the fuselage, the average coverage figure obtained for all the models was 70 per cent. It is reasonable to assume that in most cases a coverage figure of 70–90 per cent can be obtained for top and bottom antennas by using some vertical directivity.

For nose and tail antennas with cardioid patterns having 5-db gain and 3-db cable loss, the best coverage figure obtainable theoretically is 60 per cent. This is based on ±30 degrees coverage which limits the gain to 5 db. This figure can be improved somewhat by using antennas with sharper cutoff characteristics in the overlap region.

If automatic switching is employed, the circuitry for nose and tail or top and bottom antennas may have to be different.

B-(2). *Dual Antennas Connected through a Phase Shifter.* A feasible system (using top and bottom antennas) is to use one low-speed phase shifter to sweep the lobes past the ground radar at a rate to exceed the rotation rate of the ground radar antenna. Unfortunately the coverage is uncertain at all values of ϕ and the uncertainty is greatest at $\theta = 90$ degrees, which is the region of maximum interest.

B-(3). *Common Antenna Systems for DME and Beacon.* Probably the best multiple-antenna solution to the beacon problem is one consisting of radiators on the nose and tail, producing interference off the wings; while the best multiple-antenna solution for the DME problem is a system consisting of top and bottom antennas with an automatic switch. Unfortunately the DME does not respond favorably to the type of interference pattern favorable to beacon systems, nor is switching of antennas favorable to beacon operation.

The simplest common-antenna system is the nose and tail antennas operated in parallel. Since relative phase is not significant, a simple hybrid bridge may be used as a multiplexer. This system is desirable because of its simplicity. However, its performance suffers from cable and junction loss. Also, for DME the interference region is doubtful. Otherwise, the requirements that a common-antenna system imposes on a multiplexer are severe and might result in a device whose complexity would make the system impractical. In the event a practical multiplexer can be built, there is no basic reason why separate additional filtering cannot be employed to enable the antennas in a multiple-antenna system to be tied together over one frequency band and operated separately over an adjacent band.

Acknowledgment

The writers express their appreciation to A. G. Kandoian for over-all supervision of this work, to W. Spanos for assistance in taking and analyzing the pattern data, and to A. J. Lombardi for mechanical design of the model mount. Also helpful were conferences with Henry Senf and Walter Pike of A.N.D.B. and with L. J. Chu of the Massachusetts Institute of Technology.

A Preliminary Survey of Tropospheric Refractive Index Measurements for U. S. Interior and Coastal Regions*

C. M. CRAIN, J. R. GERHARDT, AND C. E. WILLIAMS†

Summary—**This paper summarizes the results of approximately 700 tropospheric refractive index soundings carried out between July, 1952 and December, 1952, using aircraft-mounted direct-reading refractometers. Extensive observations up to heights as great as 25,000 feet have been made in the southern Ohio region and off the U. S. East and West Coasts. Preliminary statistical analyses on the occurrence of standard and non-standard average gradients are presented together with qualitative meteorological correlations. The existence of large scale index variations near certain cloud boundaries is confirmed and these are tentatively explained in terms of the contrast between the free air refractive index and that of a vertically moving air current having a refractive index representative of some lower atmospheric level.**

Introduction

THE PRIMARY PURPOSE of this paper is to present a summary of the results of numerous measurements of index-of-refraction profiles over certain U. S. coastal and interior regions. As some 700 of these profiles have been measured at the various locations only a few of the actual profiles are presented here. A complete compilation of the profiles has been made elsewhere.[1] The measurements herein reported include 81 profiles obtained near Dayton, Ohio between July and October 1952, 193 profiles obtained off the Washington Coast in August and September, 1952, 146 profiles obtained off the California Coast in October, 1952 and 259 profiles obtained off the coast of Long Island and New Jersey in November and December, 1952. Except for the measurements taken in the Dayton area the measurements were made with the same refractometer and procedure as previously reported.[2,3]

The profiles for the Dayton area were obtained by the Propagation Unit of the Wright Air Development Center. The refractometers were mounted in a C-46 and in a B-17 aircraft in essentially the same manner as previously described.[2] They were used in fundamentally the same way except that the metering circuit design was such that one could measure on a 50-, 100-, or 200-N-unit scale anywhere over a 400-N-unit range. Changes in the recording pen position were accomplished electrically in calibrated steps rather than with the mechanical arrangement used in previous tests.

The profiles shown for the coastal areas were obtained by the same means and using the same refractometer reported in reference 2 except that the refractometer was carried aloft in a B-25. These measurements were sponsored by the Air Force Cambridge Research Center.

* Research sponsored by Air Force Contracts AF 18(600)-113 and AF 19(604)-494.

† University of Texas, Austin, Texas.

[1] C. M. Crain, "Refractometer measured tropospheric index-of-refraction profiles," vol. I and II Electrical Engineering Research Lab. Rpts. no. 6-02 and 6-03, University of Texas, 1953.

[2] C. M. Crain and A. P. Deam, "An airborne microwave refractometer," *Rev. of Sci. Instr.*, vol. 23, pp. 149–151; April, 1952.

[3] C. M. Crain, A. P. Deam, and J. R. Gerhardt, "Measurement of tropospheric index-of-refraction fluctuations and profiles," Proc. I.R.E., vol. 41, pp. 284–290; February, 1953.

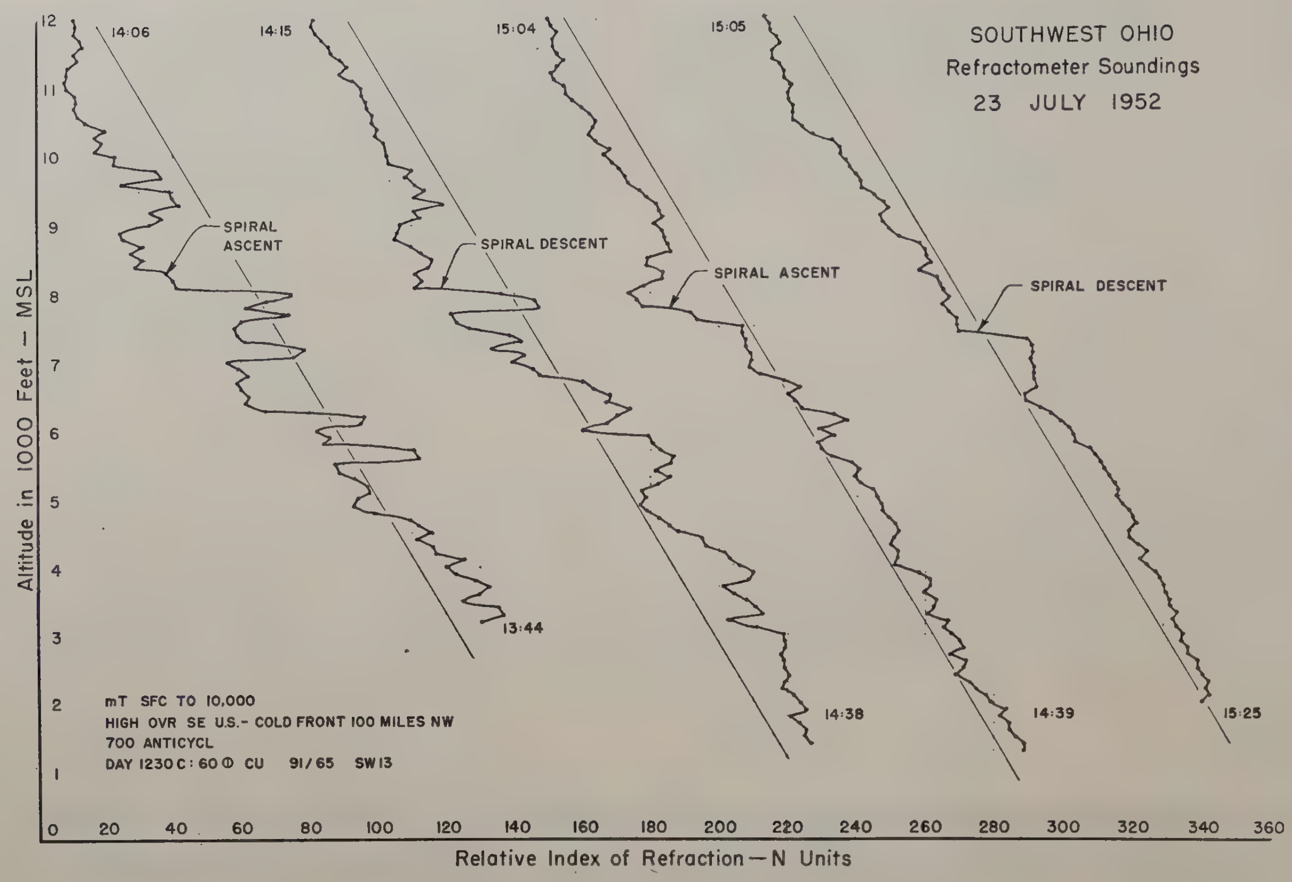


Fig. 1—A series of four refractometer soundings taken over the period from 1344 to 1525 CST, July 23 in the southwest Ohio area. These data illustrate a significant decrease in refractive index fluctuations at nearly all levels in the first 10,000 feet during the approximate 80 minute measurement interval. The weather code is translated as follows: maritime tropical air mass from surface to 10,000 feet; high pressure area over Southern U. S. with a cold front 100 miles to the northwest; anticyclonic circulation at 700 millibars; weather report for Dayton, 1230 CST gives scattered cumulus clouds at 600 feet, temperature and dewpoint of 91 and 65 degrees F respectively and a surface wind of 13 knots from the southwest.

For all the measurements reported the planes' rate of ascent and descent was in the range of 500 to 1,000 feet per minute and the planes' indicated air speeds were in the range 120 to 150 mph for the C-46 and 150 to 200 mph for the B-17 and B-25. Although the rate of ascent and descent and air speed varied from sounding to sounding, the rates were held reasonably constant during each sounding.

In making the soundings the plane followed a spiral path up or down over a fixed location or flew along a slant path between two locations. In making the spiral ascents and descents the pilots were instructed to maintain the same ground reference. No check was possible for the overwater soundings; however, little drift in the ground reference was noted in the overland flights. The radii of curvature for the various spirals shown lay in the range of approximately 2 to 4 miles.

The individual soundings shown have been replotted from the original Esterline-Angus meter recordings. The height interval between plotted points is either 100 or 200 feet, and each point is a qualitative average of the refractive index records for a 50- or 100-foot variation around the point. Certain exceptions to this plan have been made where large amplitude fluctuations have occurred within height intervals of less than 100 feet, but no effort has been made to duplicate the detailed smaller amplitude index variations found in the original recordings. An analysis of the fine detail characteristics of thirty-four of the various profiles has been made and reported elsewhere.[4]

Discussion of Profiles

1. *Southwestern Ohio*

The chief general characteristics of the profiles obtained were the persistent occurrence of elevated layers between about 4,000 and 7,000 feet MSL. They varied greatly in amplitude and elevation from day to day and occasionally varied appreciably from sounding to sounding during the same day.

The occurrence of elevated layers appeared to be characterized in general by two visible phenomena, i.e., sharply defined haze layers and/or cloud boundaries. Examination of wet- and dry-bulb temperature profiles obtained simultaneously, by means of a wet- and dry-

[4] C. E. von Rosenberg, C. M. Crain and A. W. Straiton, "Airborne refractive index fluctuations as recorded by an airborne microwave refractometer," Electrical Engineering Research Lab. Report no. 6-01, University of Texas, 1953.

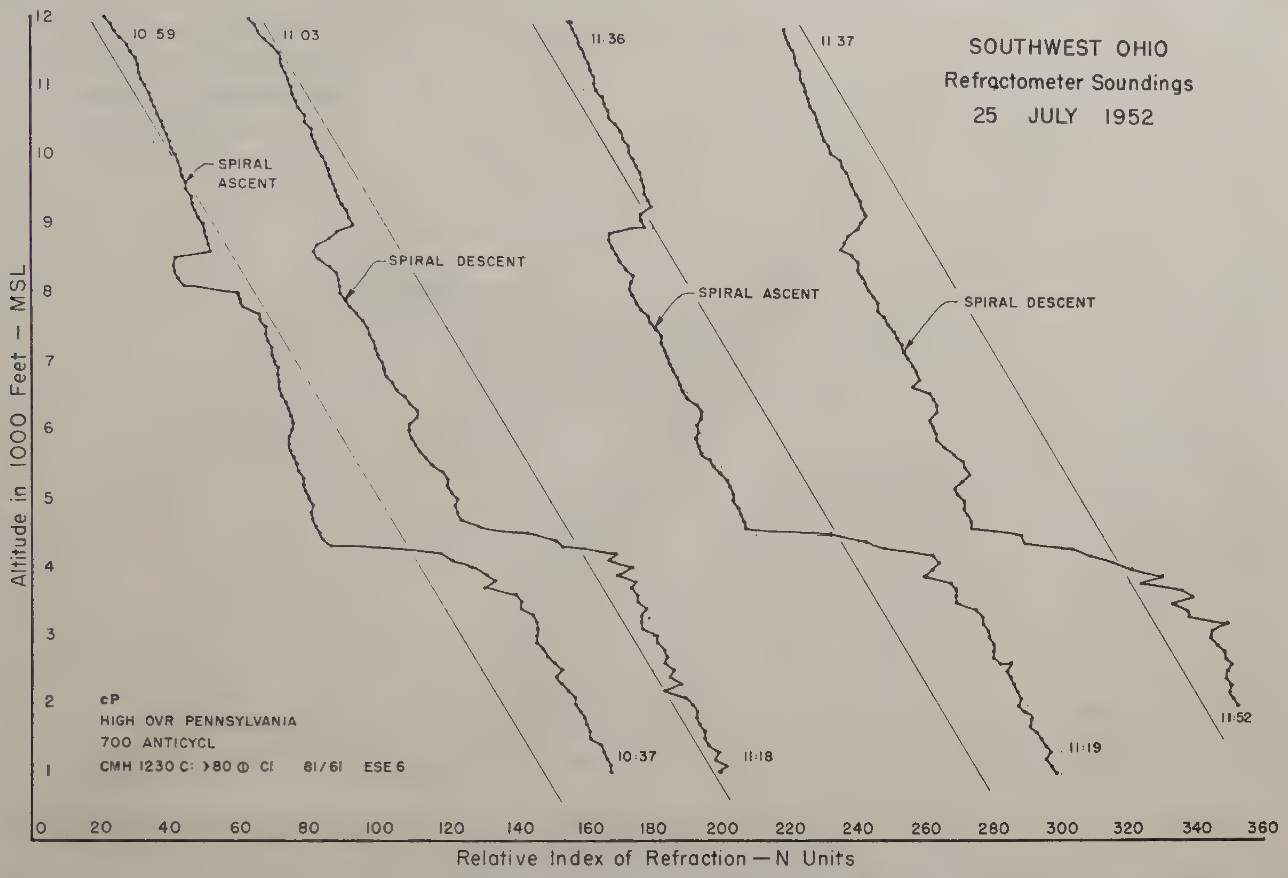


Fig. 2—A series of four refractometer soundings taken over the period from 1037 to 1152, July 25 in the southwest Ohio area. These soundings show a marked superstandard layer probably representing the boundary between the lower cP air and the overlying mT air as well as a small substandard layer within the mT air mass. The weather station call letters, CMH, refer to Columbus, Ohio.

bulb psychrograph,[5] revealed that a dry bulb temperature inversion of 1 to 3 degrees was normally associated with the stronger layers. Temperature data were not available for all soundings.

The rapidity with which the shape of the profiles may vary is apparent in Fig. 1. These profiles were obtained on July 23 over a two hour interval. On the first ascent starting at 13:44 EST, large index variations from the standard were observed from 3,000 to 10,000 feet. On the last descent beginning at 15:05 EST the large variations were no longer present and a well defined layer had formed at 7,300 feet.

From a brief examination of the weather summaries for the days during which profiles were obtained, it was found that there was a fairly even division of observations under maritime tropical (*mT*) and continental polar (*cP*) conditions. In general, soundings made under *mT* conditions showed a relatively more uniform height variation of refractive index than those taken under *cP* conditions, where there was generally a definite superstandard layer representing a frontal surface aloft. Fig. 2 is an example of commonly observed profiles under *cP* conditions, i.e., a substandard layer a few thousand feet above the large superrefracting layer. It would appear that such a condition might well be represented by a subsiding and consequently heating and drying of the *mT* air mass above the *cP* frontal interface.

2. *West Coast*

The measurements taken during this phase of the refractive index measurement program were made during 28 flights between August 11 and September 2, 1952 off the coast of Washington State and 17 flights between October 6 and October 19, 1952 off the California Coast near San Francisco. The measurements at both locations were taken while flying on a westerly path from the coast line out over the ocean. In addition, a few flights were made in which soundings were taken from the southern to northern state boundaries along the Washington Coast. Flights were made for distances out to some 150 miles from the coast with individual soundings from approximately 1,000 feet up to 10,000 or 15,000 feet above the water.

Once again, as in Southwest Ohio, the most significant general feature of these soundings was the persistent occurrence of elevated layers. The most commonly observed layers were associated with a stratus cloud layer in the first few thousand feet above the surface. A review of the general meteorology of the U. S. West Coast area indicates that this region is characterized by

[5] F. J. Overcash, "Index of refraction measurements with an airplane psychrograph," Wright Air Development Center Technical Report no. 6621, August, 1951.

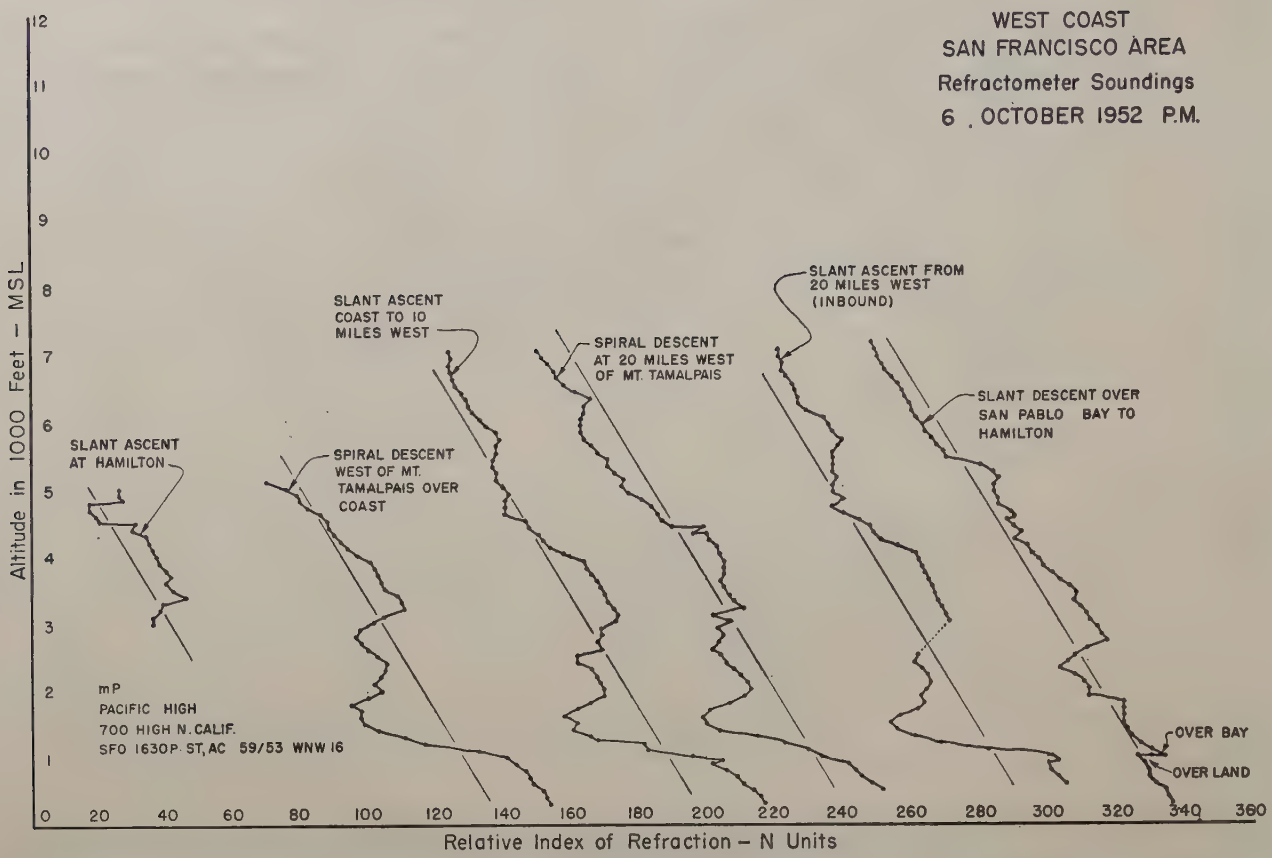


Fig. 3—A series of six refractometer soundings taken during the afternoon of October 6 for heights up to 7,000 feet and for distances out to some 20 miles overwater at a location on the California coast just north of San Francisco. These data illustrate a well marked superstandard marine layer existing some 1,000–1,500 feet over the Pacific coastal water. Note that this layer is rapidly broken up as we travel inland from the coast. SFO are the station call letters for San Francisco and mP represent a maritime Pacific air mass.

the presence of a persistent stratus cloud layer lying either throughout or in the upper portion of a cool, moist so-called marine layer immediately above the water surface. Above this marine layer there is found, as a result of marked subsidence in the eastern portion of the semi-permanent Pacific high pressure system, a warm and very dry layer. Such a superposition of layers generally produces a well marked temperature inversion and moisture gradient leading to a superstandard layer at some elevation in the first few thousand feet above the water. Fig. 3 shows a typical example of profiles in which a moderate superrefracting layer is associated with the inversion layer. Note that in the sounding taken over San Pablo Bay the layer is absent. Figs. 4, 5 and 6 show profiles obtained on August 28, 29 and 30, respectively. These are examples of the large day to day changes which may occur in the profile structure. Soundings on the 28th prior to an *mP* cold front passage showed some of the most irregular refractive index features observed while the data for August 29 following the frontal passage showed exceptionally uniform lapse rates, except for a moderate layer at 7,000 feet which may have represented the frontal surface aloft. On August 30 the substandard layer at 5,300 feet was so intense that the index of refraction was the same at 7,000 feet as it was at 3,500 feet, instead of having the standard value of about 42 N-units less. Fig. 6 also shows an example of how the profile structure may vary with distance and/or time.

3. *East Coast*

The measurements taken during this phase of the refractive index sounding program were made during 9 flights between November 11–21, 1952 on a generally northeasterly heading along a line from Long Island to Nantucket Island and during 16 flights between December 4–17, 1952 on an easterly heading out from a location on the northern New Jersey coast. Both flight schedules included data for heights up to 15,000 feet and for distance out to 150 miles overwater. Once again the individual soundings were characterized by the persistent occurrence of non-standard layers, although they were for the most part significantly reduced in amplitude from the previously described observation programs. The primary cause here is quite probably a simple climatic factor where it would be anticipated that winter air masses would have significantly lower mean moisture contents and hence less likelihood of intense moisture gradients. The East Coast data probably come closer to be representative of the type of soundings previously observed in the vicinity of southern Ohio than those over the West Coast, particularly in

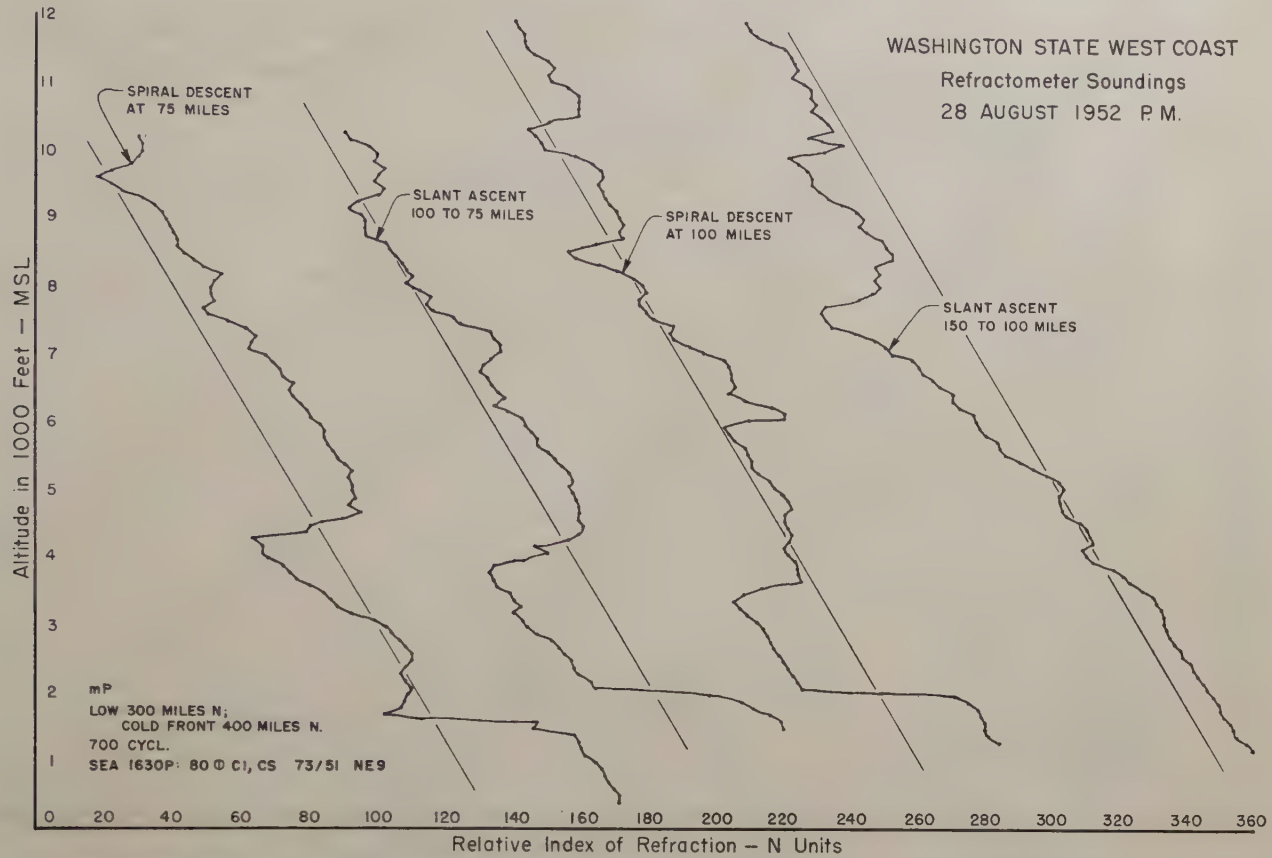


Fig. 4—A series of four refractometer soundings taken during the afternoon hours of August 28 at distances from 75 to 150 miles off the Washington coast line. Note that the well marked marine layer at about 2,000 feet and the substandard layer at about 4,000 feet for the first three soundings no longer existed at 150 miles. There is some evidence here that these layers approach each other as distance offshore increases. SEA are the station call letters for Seattle.

the upper levels, since the Atlantic coastal air masses do not give rise to the inversion layers typical along the Pacific Coast and the main atmospheric flow is from west to east. The effect of the modification of the overland index structure by water vapor and heat transport occurring at the ocean surface probably does not extend to more than 2,000–3,000 feet for the overwater travel involved in these series of measurements.

The strongest layer observed over the November–December, 1952 observation period occurred on November, 21 when a warm front was reported as being some 150 miles west of LaGuardia. The layers shown in Fig. 7 at an altitude of between 4,000–5,000 feet over Long Island thus could well have represented the position of the frontal surface at these locations.

Summary of Profile Data

A summary of the approximate 700 index-of-refraction profiles taken during the previously described measurement programs is presented in Fig. 8. This figure gives the distribution of the intensity of the observed elevated layers for each of the sounding locations, no attempt being made to take into account differences in time or position of the sounding within the general sounding area. Using as a criterion the maximum change in the refractive index profile in N-units for any 200 foot-height interval, we see for the July and August measurements over southern Ohio that 50 per cent of all the soundings showed "layers" having refractive index changes of the order of 25 N-units in 200 feet. On the other hand, the New Jersey coastal area in December gave an approximate 7 N-unit change for the 50 per cent level. On a qualitative basis, this figure thus shows the change from a summer to a winter climate on the vertical structure of the troposphere but it should be kept in mind that the data are insufficient to distinguish between sounding areas or to establish quantitative climatic variations.

Observations through Clouds

Although there has been no co-ordinated efforts to obtain refractive index data in and near clouds, several such observations have been made and free air to cloud refractive index differences of as large as 40–50 N-units for isolated cumulus type clouds have been noted. Except for the West Coast stratus, flights through other types of clouds have not been made. Representative examples of these recordings are shown in Fig. 9. The first part of this figure shows a segment of the original Esterline-Angus recording of a flight through an isolated cumulus cloud at an elevation of about 7,700 MSL over southeast Ohio on July 23, 1952. Note that the total index change through the cloud of about 10 N-units is almost exactly the same as the change

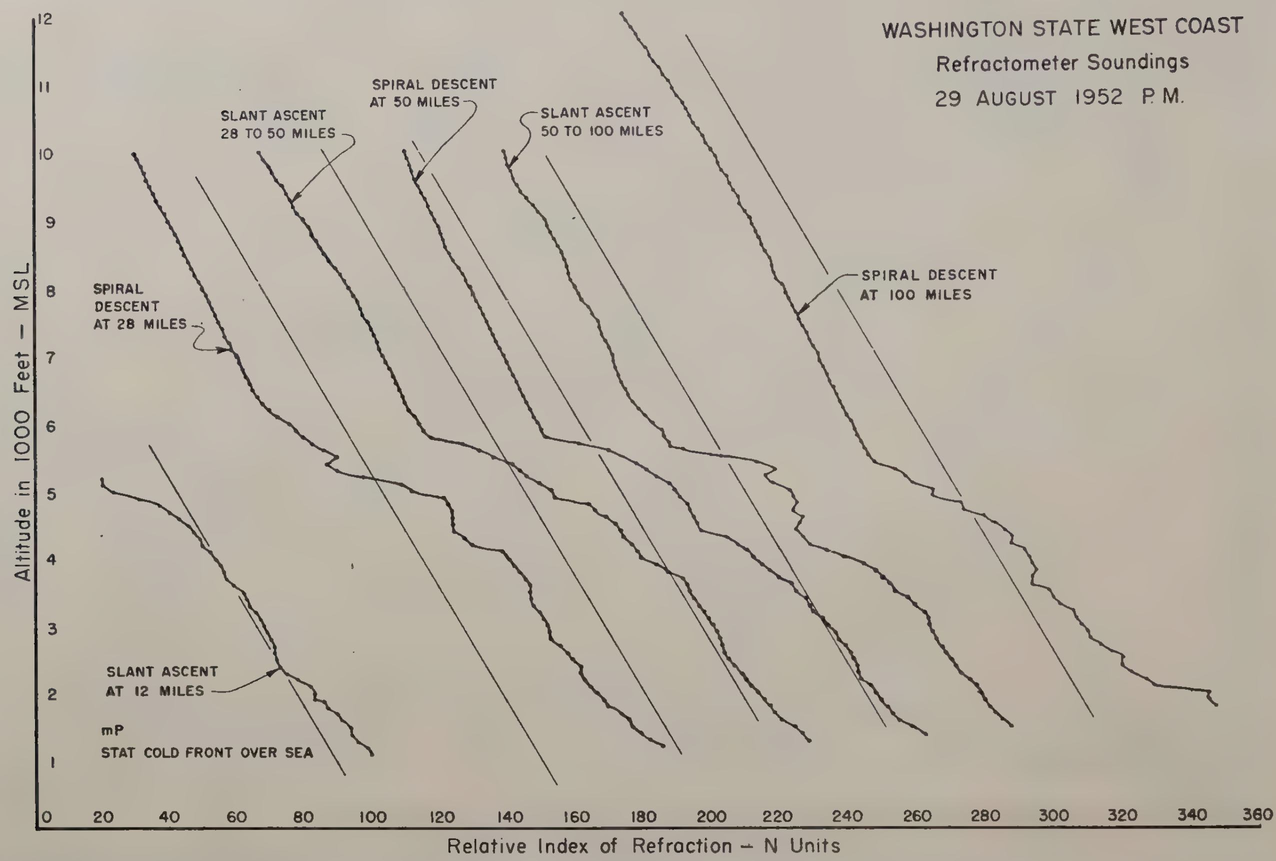


Fig. 5—A series of six refractometer soundings taken during the afternoon hours on August 29 over the offshore distances from 12 to 100 miles for the Washington coast area. Note that these soundings taken some 24 hours later than those shown in Fig. 4 following the passage of an mP cold front show relatively few irregularities or fluctuations. The superstandard layer at about 5,000 feet may represent the frontal position aloft.

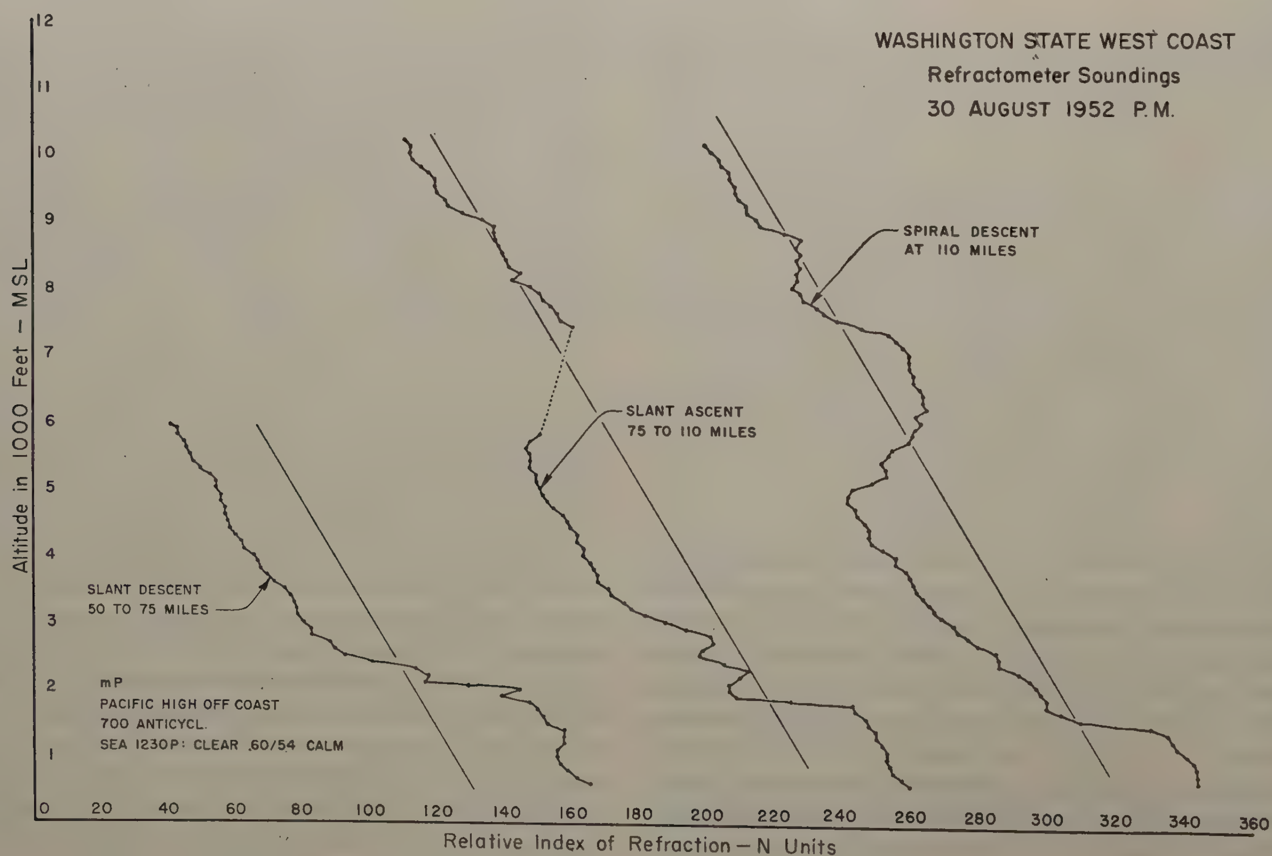


Fig. 6—A series of three soundings taken on August 30 some 24 hours after those of Fig. 5 and showing very large superstandard and sub standard gradients. Note once again that the marine layer decreases in elevation as distance from shore increases.

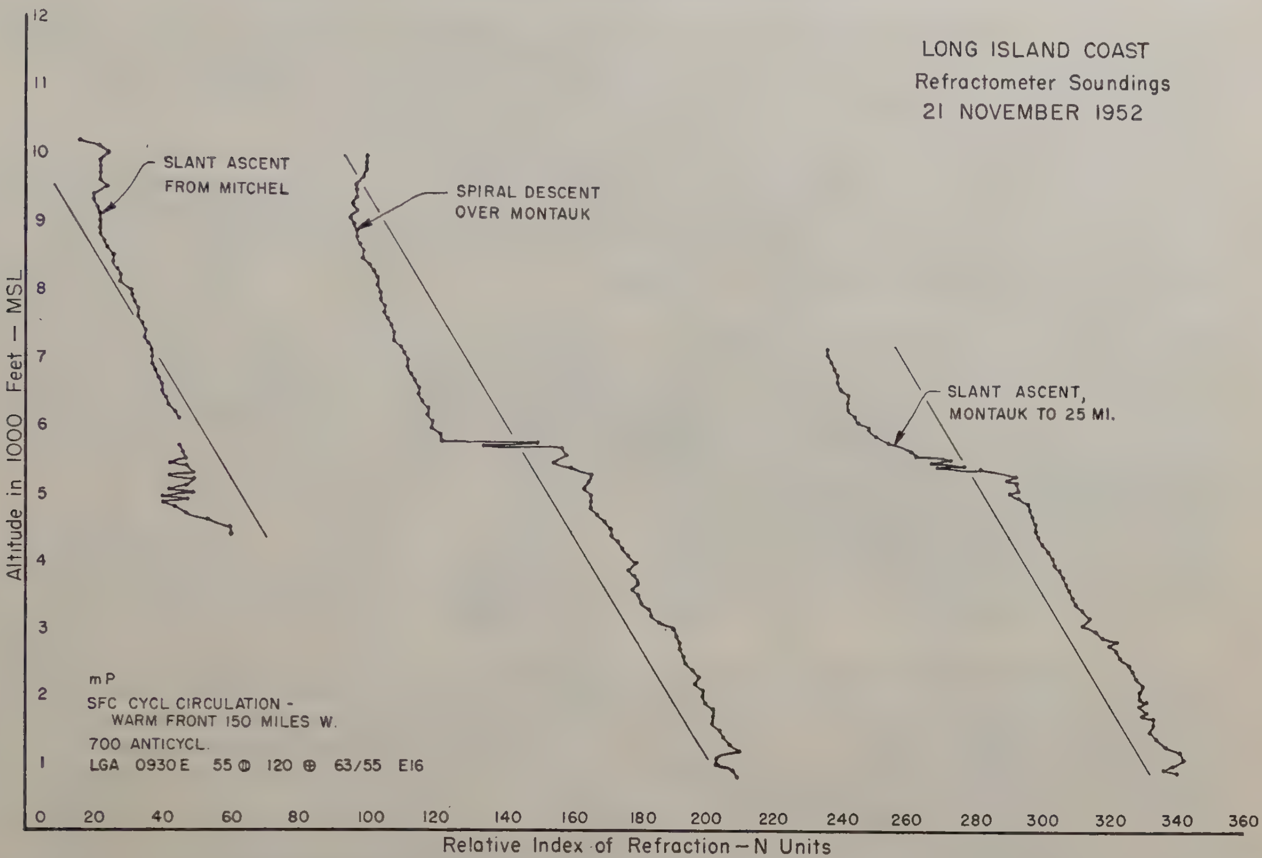


Fig. 7—A series of three refractometer soundings taken during the forenoon hours of November 21 over Long Island, New York. The strong superstandard gradient at about 5,500 feet probably represents the position of the warm front over Long Island. IGA are the call letters for LaGuardia Airport, New York.

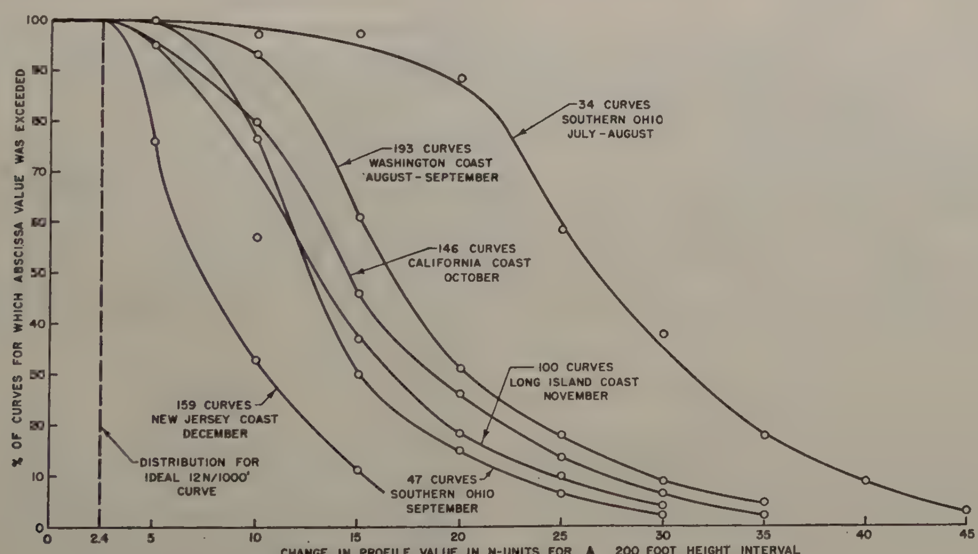


Fig. 8—Distribution curves for 679 index-of-refraction profiles.

through the superstandard layer at 7,300–7,400 feet and it would appear reasonable to believe that the saturated air within the cloud at 7,700 feet has as its source the air below the superstandard layer at 7,300 feet. A more violent demonstration of this effect is shown in the second part of Fig. 9 where flights were made through several small cumulus clouds during a descent from 12,000 feet to the surface. Here we can note air to cloud index changes of up to 30 N-units. Once again a strong superstandard layer exists below the cloud at 6,200 feet. This layer incidentally is partially hidden on the recording due to the fact that it was necessary to change reference levels at that point to keep the recording on the chart paper. The third part of this figure shows a flight through an isolated cumulus located a few miles on-shore along the Washington coast line. Note that the index change here was over 45 N-units and that although no intense superstandard layer existed below this level, the refractive index within the cloud is representative of that from the marine layer over the Pacific. Inasmuch as there is a small coastal range here going up some 2,000 feet from the water's edge, vertical currents

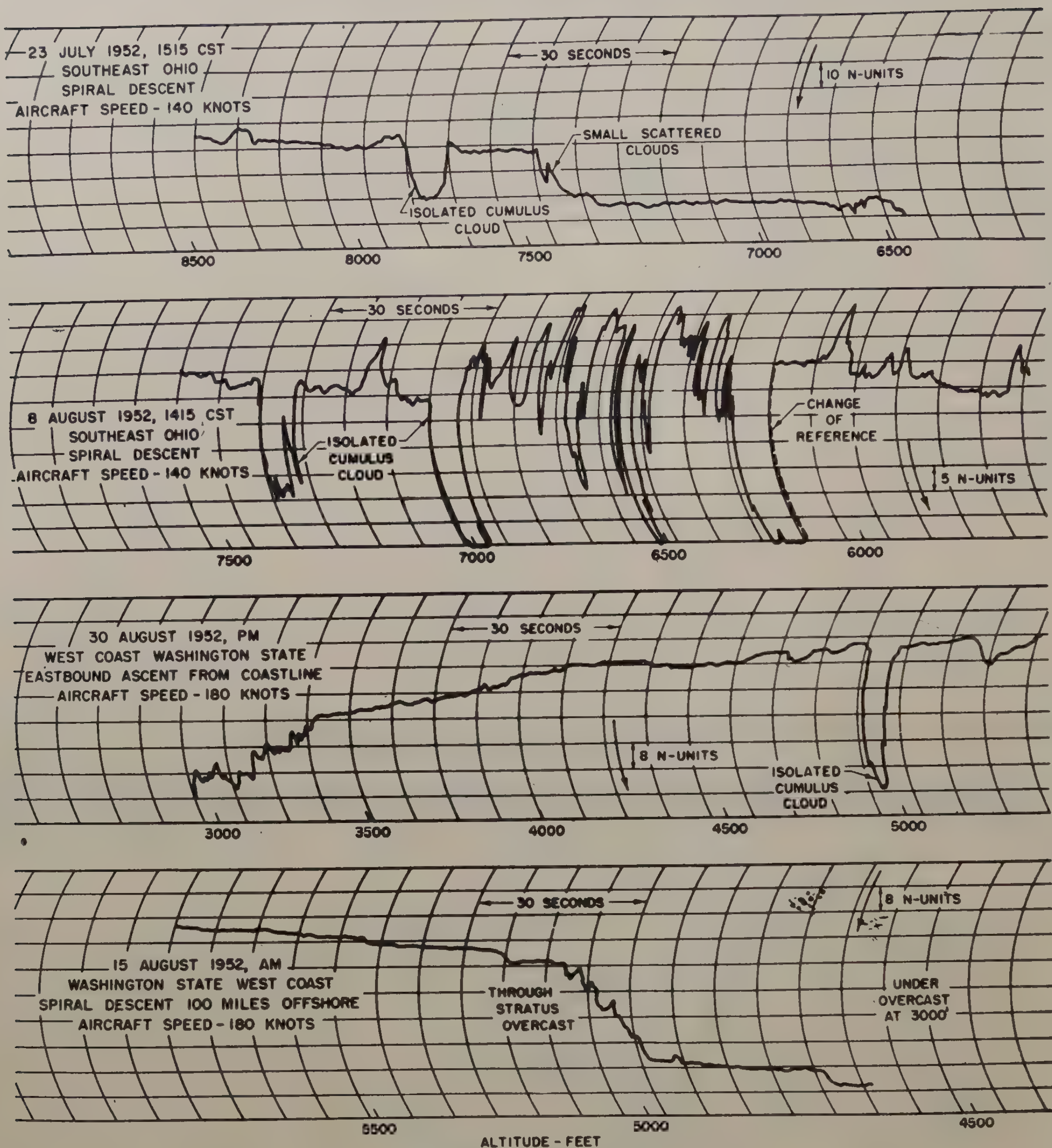


Fig. 9—Refractive index recordings through various cumulus and stratus type clouds.

could easily rise to the cloud observation height of about 5,000 feet. The fourth part of this figure illustrates a flight through the Pacific stratus layer. It should be noted that the top of the stratus and the superrefracting layer coincide at about 5,000–5,200 feet. From that point down to 3,000 feet, the base of the stratus, and down further to an altitude some 100–200 feet above the water there were no further superstandard gradients. Since this same superstandard layer had been observed, without visible stratus clouds, it is apparent that the top of the stratus is coincident with the layer representing the moisture gradient between the marine layer and the overlying warmer and dryer layer. Flights through the lower boundary of the stratus produced no such contrast and we can thus be assured that the moisture gradient between free air and stratus within the marine layer is small, i.e., that the marine layer is essentially saturated throughout its depth. It would also further appear as a result of these observations that the effect on refractive index due to liquid cloud droplets is small. The entire explanation of this cloud index increase over free air is, however, not completely settled. Measurements with aircraft psychrographs through cumulus clouds over the Caribbean have shown air to cloud moisture gradients of no more than 3–4 millibars.[6] Values of up to 10–12 millibars would be required to explain index changes of up to 40–50 N-units. A conclusive test of the rising air columns concept would appear to be that of measuring index changes at levels immediately below the cloud and it is anticipated that such measurements will be made in the near future.

[6] J. S. Malkus, "Some results of a trade cumulus cloud investigation," Woods Hole Oceanographic Institution Technical Report no. 23, May, 1953. Unpublished manuscript.

Some Stochastic Problems in Wave Propagation—Part I

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Summary—The effect of random height variations associated with a conducting surface upon the characteristics of reflected wave energy is ascertained by the methods of physical optics. Average received power, its variance, angular and frequency power spectra, and the field correlation pattern are determined in terms of the statistical parameters of the surface.

Volume type problems are treated by ascertaining the effect of refractive index fluctuations within a slab upon an emergent wavefront, and then generalizing to a continuous medium.

The results are applied to various problems encountered in tropospheric and ionospheric wave propagation.

Introduction

WE SHALL INVESTIGATE, in this paper, some of the characteristics of wave energy which has been exposed to various types of random processes. Since the parameters governing these processes are specified statistically, only a statistical specification of the resultant properties of the wave energy is expected. In general it is helpful to know the distribution function, frequency power spectrum, and space correlation pattern of the received wave energy.

The physical significance of problems of this sort arises from the fact that in many situations our transmitted waves encounter variations in medium properties whose physical causes are so complex that a statistical description of the fluctuations is all that is possible. Random effects upon propagation can be divided into two rough classes: surface effects, and volume effects. In the former category is included the modification in wave characteristics produced by reflection from a conducting screen whose surface co-ordinates are random functions of time. Examples of such a screen would be the surface of the sea, the ionosphere at the level of reflection, or a rough ground (if the time averages taken here are viewed as ensemble averages). The second category refers to the effects which accompany propagation through continuous media whose (wave) properties are a random function of position and time. The applications to electromagnetic wave propagation in the troposphere and in the ionosphere will be discussed.

Previous work along these lines includes a discussion by Bergmann[1] of some of the volume effects associated with line of sight propagation in a fluctuating medium, utilizing geometric optical methods. Booker, Ratcliffe, and Shinn[2] have treated the diffraction of plane waves by a random screen. Several of the results herein are generalizations of their theorems on the correlation in the diffracted wavefront. Rice[3] has given a perturbation treatment of reflection from a slightly rough, two-dimensional surface, while Ament[4] has given a rigorous formulation of reflection of plane electromagnetic waves from a one-dimensional screen, but has found only an approximate evaluation to be possible.

In this paper the methods of physical optics are used throughout. This precludes obtaining information about wave polarization, but enables a relatively straightforward formulation and solution of many problems for which general considerations indicate physical optics to be a very good approximation to the true wave picture. Surface problems are treated here, while volume problems will be taken up later.

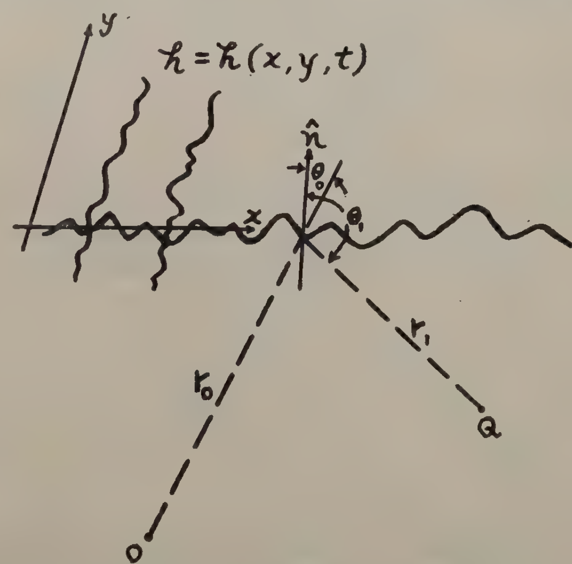

Fig. 1—Physical optics formulation of surface reflection.

Surface Problems

The Physical Optics Formulation of Surface Reflections

Consider first the reflection of waves from a surface $h(x, y, t)$ corrugated in one dimension, i.e., assume the random variations in the height of the surface to be a function of a single co-ordinate x, and of time. The general formula employed in physical optics for the calculation of field intensity produced by reflection of a harmonic disturbance is:[5]

$$E_Q = \frac{kE_0}{4\pi} \int_S e^{ik(r_0+r_1)-i\omega_0 t}(\cos\theta_0 - \cos\theta_1)\,\frac{dS}{r_0 r_1}, \qquad (1)$$

where E_0 is the amplitude at unit distance from the source, $k = 2\pi/\text{wavelength} = \omega_0/c$, and the meaning of the other symbols is indicated in Fig. 1. The usual sta-

† National Bureau of Standards, Washington 25, D. C.

[1] P. G. Bergman, "Propagation of radiation in a medium with random inhomogeneities," *Phys. Rev.*, vol. 70, p. 486; 1946.

[2] H. G. Booker, J. A. Ratcliffe, and D. H. Shinn, "Diffraction from an irregular screen with application to ionospheric problems," *Phil. Trans. Roy. Soc. A*, vol. 262, p. 579; 1950.

[3] S. O. Rice, "Theory of Electromagnetic Waves," p. 351, N.Y.U. Symposium, Interscience Publishers; 1951.

[4] Ament, W. S., "Toward a theory of reflection by a rough surface," Proc. I.R.E., vol. 41, p. 142, January; 1953.

[5] B. B. Baker and E. T. Copson, "Mathematical Theory of Huygen's Principle," Clarendon Press, Oxford; 1950.

tionary phase approximation in the y-co-ordinate may be employed since by hypothesis no random fluctuations enter here. If one also neglects the variation of the distance and obliquity factors in (1) over the region of the surface from which significant contributions are received, then (1) reduces to:

$$E_Q = \frac{E_0}{D_1 D_2}\sqrt{\frac{k\overline{D}}{\pi}\cdot\frac{\cos\theta}{2}}\int_{-\infty}^{\infty} e^{ik(\phi_1+\phi_2)-i\omega_0 t}dx \tag{2}$$

where

$$\phi_1=\sqrt{(D_1\sin\theta+x)^2+(D_1\cos\theta+h)^2}$$

$$\frac{1}{\overline{D}}=\frac{1}{2}\left(\frac{1}{D_1}+\frac{1}{D_2}\right).$$

$$\phi_2=\sqrt{(D_2\sin\theta-x)^2+(D_2\cos\theta+h)^2}$$

$h=h(x,t)$ is the random height variation about the mean height of the surface. The remaining symbols are explained in Fig. 2. It should be noted that these expres-

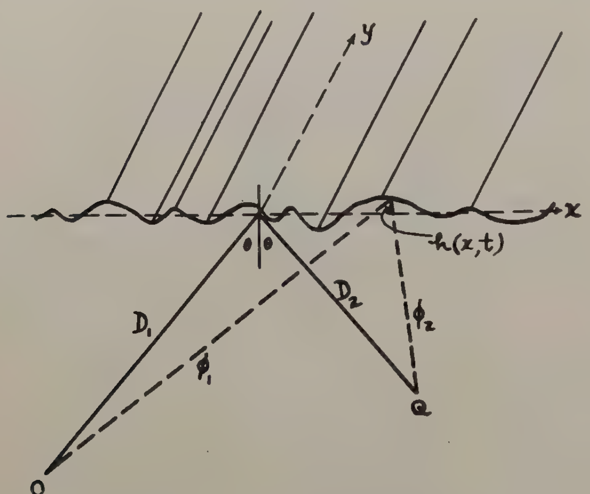

Fig. 2—Application to a surface corrugated in one dimension.

sions for the phase imply a lack of shielding of any element of the surface by any other element. This neglect of shadowing will be further discussed later. Expanding the phase to second-order terms, (2) becomes:

$$E_Q = \frac{E_0}{D_1 D_2}\sqrt{\frac{k\overline{D}}{4\pi}}\cos\theta$$

$$\cdot\int_{-\infty}^{\infty} e^{(ik/\overline{D})\cos^2\theta\cdot x^2+2ikh\cos\theta-i\omega_0 t}dx, \tag{3}$$

where a phase term $(kh^2/\overline{D})\sin^2\theta$ has been neglected, since it is comparable to $2kh\cos\theta$ only at extreme grazing incidence. To compute the average value of the reflected power multiply this expression by the complex conjugate of the accompanying magnetic field and take half the time average of the real part:

$$\overline{P}_Q = \frac{E_0^2}{Z_0 D_1^2 D_2^2}\cdot\frac{k\overline{D}}{8\pi}\cos^2\theta\cdot \mathrm{Rl}\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}dt$$

$$\cdot\iint_{-\infty}^{\infty} e^{(ik/\overline{D})\cos^2\theta(x^2-x'^2)+2ik\cos\theta[h(x,t)-h(x',t)]}dx,dx', \tag{4}$$

where Z_0 is the impedance of space. Some assumptions regarding the properties of the random variable h must now be made. Assume that h is normally distributed with variance $\overline{h^2}$ at all points, and independent of time. Next the distribution function of the difference of the h's must be specified. If we assume that the joint distribution of the heights at any two positions is of the bivariate normal type then the distribution of this difference will also be normal with a variance given by:

$$\overline{(\Delta h)^2}_{x-x'} = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}dt\{h(x,t)-h(x',t)\}^2 \tag{5}$$

$$= 2\overline{h^2}\{1-\rho(x-x')\}, \tag{5a}$$

where

$$\rho(x-x') = \frac{1}{\overline{h^2}}\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}dt\cdot h(x,t)\cdot h(x',t) \tag{6}$$

is termed the space correlation function of h, and is here assumed to depend only upon the separation of the two points. Interchanging the order of integration in (4), and utilizing the fact that an average over time of a function of a random variable may be replaced by an average over its probability distribution:

$$\frac{1}{2T}\int_{-T}^{T}dt\cdot e^{2ik\cos\theta\cdot[h(x,t-h(x',t)]}$$

$$= \int_{-\infty}^{\infty}d(\Delta h)\cdot\frac{e^{-(\Delta h)^2/2\overline{(\Delta h)^2}_{x-x'}}}{\sqrt{2\pi\overline{(\Delta h)^2}_{x-x'}}}\cdot e^{2ik(\Delta h)\cos\theta}$$

$$= e^{-2k^2\overline{(\Delta h)^2}_{x-x'}\cos^2\theta}. \tag{7}$$

One obtains after the transformation $\xi=x-x'$,

$$\overline{P}_Q = \frac{E_0^2}{Z_0 D_1^2 D_2^2}\cdot\frac{k\overline{D}}{8\pi}\cos^2\theta\iint_{-\infty}^{\infty}dxd\xi\cdot e^{(ik/\overline{D})\cos^2\theta(2x\xi-\xi^2)}$$

$$\cdot e^{-4k^2\cos^2\theta\cdot\overline{h^2}[1-\rho(\xi)]}, \tag{8}$$

$$= \frac{1}{2Z_0}\left(\frac{E_0}{D_1+D_2}\right)^2 \tag{8a}$$

by the Fourier integral theorem, since $\rho(0)=1$. Consequently the average total energy received is identical with that resulting from a specular reflection, although the angular distribution of this energy may be considerably different. In reality this result may be obtained directly from (4) provided only that the turbulence is homogeneous, i.e., independent of x.

To obtain the angular power spectrum one must adopt a specific function for $\rho(\xi)$. If the latter is of the form $e^{-\xi^2/l^2}$ then letting $a=4k^2\overline{h^2}\cos^2\theta$, one obtains for $p(x)$ [with $\int p(x)dx=1$ normalization]:

$$p(x) = \frac{k\cos^2\theta}{\pi\overline{D}}\int_{-\infty}^{\infty}d\xi\cdot e^{(ik\cos^2\theta/\overline{D})(2x\xi-\xi^2)}$$

$$\cdot e^{-a}\cdot\sum_{n=0}^{\infty}\frac{a^n}{n!}e^{-n\xi^2/l^2}, \tag{9}$$

$$= e^{-a}\sum_{n=0}^{\infty} \frac{e^{-nx^2/l^2[1+(ng)^2]}}{\pi g\sqrt[4]{1+(ng)^2}} \cdot \cos\left[\frac{\frac{kx^2}{\overline{D}}\cos^2\theta}{1+(ng)^2} + \frac{1}{2}\arctan(ng)\right], \tag{9a}$$

where $g=\overline{D}/kl^2\cos^2\theta$, a measure of the ratio of the size of a Fresnel zone to the scale of coherence. The $n=0$ term yields the residual smooth surface reflection

$$e^{-a}\cdot\cos\left(\frac{kx^2}{\overline{D}}\cos^2\theta\right), \tag{10}$$

a result obtained by Ament[4] and others. The higher order terms contain the contribution of the scattered fields. For the usual condition $g \gg 1$, the oscillations with surface position of these contributions cease; the energy scattered from a wide range of angles then adds arithmetically at the receiver. It is instructive to define an angular power spectrum through the relation $\tan\phi = x\cos\theta/\overline{D}$. For $g\to\infty$, (9a) then yields:

$$p(\phi) = e^{-a}\delta(\phi) + e^{-a}\sum_{n=1}^{\infty}\frac{a^n}{n!}\cdot\frac{kl\cos\theta}{\sqrt{\pi n}} \cdot e^{-(k^2l^2\cos^2\theta/n)\tan^2\phi}\sec^2\phi \tag{11}$$

where we have normalized so that $\int_{-\pi/2}^{\pi/2}p(\phi)d\phi=1$. $\delta(\phi)$ denotes the fact that the residual smooth surface reflection is obtained only at the specular angle. This representation results from the operational viewpoint that the oscillations of the cosine factor yields a nonzero net contribution only for $x\cong 0$, as a result of the finite resolving power of the receiving antenna.

It is important to note that with an exponential correlation function $\rho(\xi)=e^{(-\xi/l)}$, (11) becomes:

$$p(\phi) = e^{-a}\delta(\phi) + e^{-a}\sum_{n=1}^{\infty}\frac{a^n}{n!}\cdot\frac{2kl\cos\theta}{\pi n} \cdot \frac{\sec^2\phi}{1+\left(\frac{2kl\cos\theta}{n}\tan\phi\right)^2}. \tag{11a}$$

The half-power points given by (11) and (11a) are about the same, but the scattered energy at large ϕ differs greatly. This difference is associated with the space periodicity spectrum of the fluctuations, which is the transform of the correlation function. The components of small spacing produce the wide-angle scattering, and the amplitudes of these components decay much more rapidly for a Gaussian correlation function than for an exponential one.

We are now in a position to discuss the limits of validity imposed by the neglect of shadowing in our formulation. As a first approximation one may replace the irregular surface fluctuations with rectilinear corrugations having slopes distributed about a mean value $\sqrt{\overline{h^2}}/l$. Geometric considerations then indicate that for wave energy approaching the surface at angles less than $\bar{\alpha}$, where $\bar{\alpha}=\arctan\sqrt{\overline{h^2}}/l$, there will be interception by an element of the surface different from that postulated in our previous treatment. Reciprocal considerations apply at the receiver, i.e., energy presumed to leave the surface at angles less than $\bar{\alpha}$ will generally be prevented from reaching the receiver by adjacent surface elements. Consequently, shadowing effects become significant for $(\pi/2)-\theta-|\phi|<\bar{\alpha}$, and the corresponding portion of the angular power spectrum, $p(\phi)$, loses its validity. The energy in this portion of the spectrum is redistributed through complex multiple scattering processes.

In the next section the formulation of physical optics is modified so as to extend the range of validity of our methods.

A second quantity of interest is the space correlation pattern at the receiving site. Define

$$\overline{C}_Q(s) = \frac{\lim\limits_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}dt\cdot E_Q\cdot E^*_{Q+S}}{\lim\limits_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}dt\cdot|E_Q|^2}. \tag{12}$$

To obtain E_{Q+S} replace x by $x-s$ in ϕ_2 of (2). This corresponds to a displacement s, along the transmission path. Equation (8) then becomes:

$$\overline{C}_Q(s) \sim \mathrm{Rl}\, e^{iks\sin\theta}\iint_{-\infty}^{\infty}dxd\xi\cdot e^{(2ik\cos^2\theta/\overline{D})x(\xi-(s/2)(\overline{D}/D_2))} \cdot e^{-(ik/\overline{D})\cos^2\theta(\xi^2-(s^2\overline{D}/2D_2))}\cdot e^{-a[1-\rho(\xi)]}, \tag{13}$$

$$\overline{C}_Q(s) = e^{-a[1-\rho(s\overline{D}/2D_2)]} \cdot\cos\left[\frac{k\cos^2\theta}{2D_2}s^2\left(1-\frac{\overline{D}}{2D_2}\right)+ks\sin\theta\right], \tag{13a}$$

again making use of the Fourier integral theorem. For a plane wave striking the surface at normal incidence, $\overline{D}=2D_2$, and the correlation pattern on the ground becomes identical with the correlation pattern of the field variations across the screen. This is analogous to the result obtained by Booker, Ratcliffe, and Shinn.[2] For D_1 finite, the distance scale of the two patterns differs by the geometric factor $\overline{D}/2D_2$, the ratio of the distances from the source of screen and receiver; in addition an oscillating phase factor is superimposed on the received pattern.

To summarize, these results are applicable when the usual approximations of physical optics are valid, and when shadowing effects may be neglected. Under these conditions it has been shown that the average received power retains its specular value although the angular power spectrum may be broadened considerably. In addition, the ground correlation pattern is simply related to the phase deviations produced by the surface.

Modification Necessary for Small-Scale Turbulence

It is apparent from either (9) or (11) that for small "kl", contributions are received over a wide range of

angles, corresponding to regions of the scattering surface much larger than the Fresnel zone responsible for reflection from a smooth surface. Under these conditions, both the assumption of constant source-screen vector distance and the second-order approximation to the phase made in passing from (1) to (3) become invalid. One finds upon investigation that the distance factor predominates over the phase correction in decreasing the contributions at large angles. The required modification of (3) for the worst case, $D_1 = D_2$ and normal incidence is

$$E_Q \cong \int_{-\infty}^{\infty} \frac{dx}{\left[1+\left(\frac{x}{D}\right)^2\right]^{3/2}} \cdot e^{ik[x^2/D+2h]}. \tag{14}$$

In passing from (4) to (8) one may set $x' = x$ in the denominator factor, since the range of ξ over which significant contributions are received is small when this type of correction is required. Consequently $p(x)$ in (9) now contains the factor $[1+(x/D)^2]^{-3}$, and integrating over all x, one obtains for the reflection coefficient when $g \gg 1$,

$$\begin{aligned}\overline{R} &= e^{-a}\sum_{n=0}^{\infty} \frac{a^n}{n!}\frac{1}{\sqrt{\pi}}\int_{-\infty}^{\infty}\frac{e^{-u^2}du}{\left(1+\frac{n}{k^2l^2}u^2\right)^3} \\ &= e^{-a}\left\{1+\sum_{n=1}^{\infty}\frac{a^n}{n!}\frac{q}{2}\left[\sqrt{\pi}\,e^{q^2}\operatorname{erfc}(q) \cdot (q^4-q^2+\tfrac{1}{4}) + q(\tfrac{3}{2}-q^2)\right]\right\},\end{aligned} \tag{15}$$

where $q = kl/\sqrt{n}$.

This expression has been plotted as a function of "kl" for various values of "a" in Fig. 3. The reflection coefficient would approach e^{-a} as "kl"$\to 0$, if this were the sole correction. As this limit is approached, however, another modification becomes necessary as a result of the failure of physical optics when dimensions of the order of and less than a wavelength are present. The concept of phase along a trajectory then fails as a result of the averaging performed by the wave in its response to the medium characteristics. Mathematically, the physical optics solution no longer satisfies the wave equation as a result of the rapid variation of h, and consequently the phase, with x. A heuristic approach to the correction required in this case consists in modifying the propagation vector so that the wave function continues to satisfy the wave equation on the average, at a plane immediately below the surface corrugations. To this end, we write

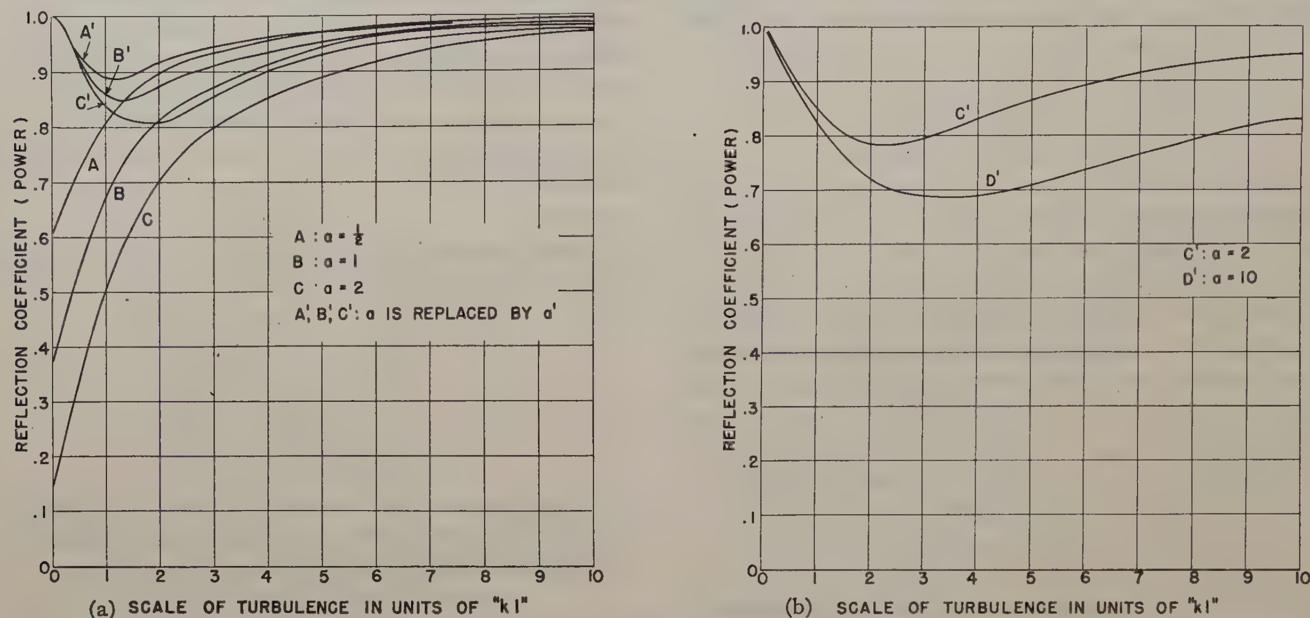

Fig. 3—Variation of received energy with roughness parameters: (a) One dimensional, (b) two dimensional.

$$\psi = e^{i\mu[z'+2h(x,t)\cos\theta]}. \tag{16}$$

The wave function can always be placed in this form of course, if we allow μ to be a function of the co-ordinates and of time. We now make the assumption that μ varies much less with time and with the space co-ordinates than does h, so that in taking the usual time average of $\psi\psi'$ [as in (7)] it may be treated as constant, simply replacing k. Then to find the appropriate average value of μ we substitute (16) in

$$\overline{(\nabla^2 + k^2)\psi} \doteq 0. \tag{17}$$

We now make the related assumption that the spatial variations of μ within a wavelength are small, so that its space derivatives can be neglected. Then since $\overline{\partial^2 h\partial/x^2} = 0$, we obtain

$$\overline{\mu^2} = k^2/1 + 4\overline{\left(\frac{\partial h}{\partial x}\right)^2}\cos^2\theta. \tag{18}$$

The required variance of $\partial h/\partial x$ may be obtained from the correlation function of h:

$$\overline{\left(\frac{\partial h}{\partial x}\right)^2} = \frac{1}{2T}\int_{-T}^{T} dt\cdot\left\{\lim_{\Delta x\to 0}\left[\frac{h(x,t)-h(x+\Delta x,t)}{\Delta x}\right]\right\}^2$$
$$= \frac{2\overline{h^2}}{l^2}, \quad \text{for} \quad \rho(\Delta x) = \epsilon^{-(\Delta x)^2/l^2}. \tag{19}$$

The zero field contour occurs now, not on the surface, but at a height z, given by:

$$e^{ikz} = e^{i\mu[2h(x,t)-z]}, \tag{20}$$

$$z = h(x,t)\cdot\frac{2\mu}{k+\mu}\cdot \tag{20a}$$

The smoothing out of the zero field contour agrees with our physical ideas on the lack of penetration of waves into crevices that are small compared to $\lambda/2\pi$.

To ascertain the effect of the phase variation upon the received power, one may simply replace k by μ in the phase integral (7). This has the effect of replacing the "a" defined in (9) by:

$$a' = a/1 + \frac{2a}{(kl)^2}\cdot \tag{21}$$

The effective degree of inhomogeneity is thus reduced by an amount which is dependent upon the scale of the corrugations. The solid curves in Fig. 3(a) give the modification of (15) produced by (21).

A word should perhaps be said at this point regarding the statement by Booker, Ratcliffe, and Shinn[2] that all components in the fluctuation spectrum (equivalent to a given correlation function) which have space periodicities less than a wavelength give rise only to evanescent waves. This result is obtained by considering a single component of completely specified amplitude and phase, i.e., a diffraction grating. But the randomness inherent in a specification of only the power spectrum of space periodicity is better described by a random array of crevices, rather than by the regularity of a grating; and since the power scattered by a single crevice does not abruptly cut off at a given dimension, this reasoning by the aforementioned appears to be inapplicable.

With (13), for the correlation pattern on the ground, modified in accordance with the variable distance attenuation, it is no longer possible to obtain a form as simple as (13a). Instead one may write:

$$\overline{C}_Q(s) = e^{(ik/2D_2)\cos^2\theta\cdot s^2 + iks\sin\theta}$$
$$\cdot\int_{-\infty}^{\infty} dx\cdot p(x)\cdot e^{(ik\cos^2\theta/D_2)xs}, \tag{22}$$

where $p(x)$ is the modified angular power spectrum. If the oscillating phase factor multiplying the integral is removed from $\overline{C}_Q(s)$, then the transform of the resultant correlation pattern gives $p(x)$. This relation is independent of any of the foregoing assumptions made in dealing with small-scale turbulence.

Since the methods employed throughout here are essentially scalar in nature, it is not possible to obtain information about polarization effects from this treatment. Qualitatively, however, one may expect initial wave polarization to be preserved when the "a'" correction discussed above is negligible. For cases in which this correction is significant, cross-polarization effects will arise as a consequence of the zero divergence condition of Maxwell's equations.

Variance of Reflected Power

In principle it is possible to determine the distribution function of the received power when the necessary information is available about $h(x, t)$. A straightforward method of accomplishing this would consist, for example, in computing all of the moments of P_Q in (4), thus determining its characteristic function which in turn suffices to uniquely determine the distribution function. Because of the exponential fashion in which the random variable h enters into the expression for P, the calculation of any given order moment entails only a knowledge of twice the corresponding order joint probability distribution of h. If one made the plausible assumption that all of these were Gaussian, then the already assumed correlation function would suffice to give the required information. Because of the complexity of the steps outlined above, only the form for the second moment will be developed here, and from this the variance will be calculated for some special cases.

$$\overline{R^2} = \frac{1}{2T}\int_{-T}^{T} dt\cdot\int_x\int_\xi \cos\left[\frac{k}{D}(2x\xi-\xi^2) + \phi(x,t) - \phi(x+\xi,t)\right]d\xi dx$$
$$\cdot\int_{x'}\int_{\xi'}\cos\left[\frac{k}{D}(2x'\xi'-\xi'^2) + \phi(x',t) - \phi(x'+\xi',t)\right]d\xi' dx', \tag{23}$$

where $\phi = 2kh\cdot\cos\theta$. The two pairs of double integrals over the space co-ordinate are next written as a fourfold integral, and the trigononometric expansions employed, to bring (23) into the form:

$$\overline{R^2} = \frac{1}{2T}\int_{-T}^{T} dt\cdot\iiiint dx dx' d\xi d\xi$$
$$\cdot\left\{\cos\frac{k}{D}(2x\xi-\xi^2)\cdot\cos\frac{k}{D}(2x'\xi'-\xi'^2)\right.$$
$$\cdot\tfrac{1}{2}[\cos(\Delta-\Delta') + \cos(\Delta+\Delta')]$$
$$+ \sin\frac{k}{D}(2x\xi-\xi^2)\cdot\sin\frac{k}{D}(2x'\xi'-\xi'^2)$$
$$\cdot\tfrac{1}{2}[\cos(\Delta-\Delta') - \cos(\Delta+\Delta')]$$
$$- 2\sin\frac{k}{D}(2x\xi-\xi^2)\cos\frac{k}{D}(2x'\xi'-\xi'^2)$$

$$\cdot \tfrac{1}{2}[\sin(\Delta - \Delta') + \sin(\Delta + \Delta')], \tag{24}$$

where

$$\Delta = \phi(x, t) - \phi(x + \xi, t); \quad \Delta' = \phi(x', t) - \phi(x' + \xi', t).$$

One now performs the time averaging in a fashion analogous to (7), taking for the distribution function of $\Delta \pm \Delta'$,

$$e^{-(\Delta\pm\Delta')^2/2\overline{(\Delta\pm\Delta')^2}}/\sqrt{2\pi\overline{(\Delta \pm \Delta')^2}}.$$

The required variance is obtained in terms of the correlation function as

$$\overline{(\Delta \pm \Delta')^2} = 2a[2 - \rho(\xi) \pm \rho(\eta) \mp \rho(\xi' - \eta) \mp \rho(\xi + \eta) \pm \rho(\xi' - \xi - \eta) - \rho(\xi')], \tag{25}$$

where $\eta = x - x'$; $a = \overline{\phi^2}$. (24) now takes the form:

$$\overline{R^2} = \frac{1}{2}\int_x\int_\xi\int_{x'}\int_{\xi'}\Big[\cos\frac{k}{D}(2x[\xi-\xi'] - 2\eta\xi' - \xi^2 + \xi'^2) \cdot e^{-1/2\overline{(\Delta-\Delta')^2}} + \cos\frac{k}{D}(2x[\xi+\xi'] + 2\eta\xi' - \xi^2 - \xi'^2) \cdot e^{-1/2\overline{(\Delta+\Delta')^2}}\Big]dxd\xi dx'd\xi'. \tag{26}$$

The integration over X, yields $\delta(\xi \mp \xi')$, and integrating over ξ' then yields:

$$\overline{R^2} = \frac{1}{2}\int d\eta \int d\xi \cdot \cos\frac{k}{D}(2\eta\xi) \cdot e^{-a[2-2\rho(\xi)-2\rho(\eta)+\rho(\xi-\eta)+\rho(\xi+\eta)]} + \cos\frac{k}{D}(2\eta\xi - 2\xi^2) \cdot e^{-a[2-2\rho(\xi)+\rho(\eta)-2\rho(\xi+\eta)+\rho(2\xi+\eta)]}. \tag{27}$$

This expression has been evaluated for the case of "a" sufficiently small so that only the linear term in the series expansion of the exponentials need be considered. Using the Gaussian form of the correlation function, one obtains:

$$\overline{R^2} \cong 1 + af,$$

where

$$f = 1 + \frac{\sqrt{1+\sqrt{1+(8/u^2)^2}}}{\sqrt{2}\sqrt{1+(8/u^2)^2}} - \frac{3}{2\sqrt{2}}\frac{\sqrt{1+\sqrt{1+(4/u^2)^2}}}{\sqrt{1+(4/u^2)^2}} - \frac{\sqrt{1+\sqrt{1+(12/u^2)^2}}}{2\sqrt{2}\sqrt{1+(12/u^2)^2}}, \tag{28}$$

where

$$u^2 = \frac{2kl^2}{D}.$$

The variance, $\overline{R^2} - (\overline{R})^2 = af$. For the usual case in which l is small compared to a Fresnel zone, the variance approaches $a\ (1 - .38u)$. For l very large, the reflection becomes essentially specular, yielding a zero variance. When u^2 approaches zero, the cross-product terms which arise from the full expansion of the exponential in (27) become negligible, permitting a simple evaluation. In this case one obtains for the variance

$$\lim_{u^2\to 0}\{\overline{R^2} - (\overline{R})^2\} = \tfrac{1}{2}(e^{2a} - 1).$$

For small "kl," of course the modifications discussed in the last section, and not taken into account here, would tend to reduce this variance. It is interesting to note that in none of these cases does one approach the constant value of variance (1/2) associated with a Rayleigh distribution.

Frequency Power Spectrum

It is frequently desirable to relate the rapidity of fluctuation of the received signal to the time characteristics of the reflecting screen. A useful way of specifying this information is in terms of the frequency power spectrum of the received signal, which is obtainable in turn from the autocorrelation function of the latter.

$$\overline{P_Q}(\tau) = \frac{1}{2T}\int_{-T}^{T} E_Q(t) \cdot E_Q{}^*(t+\tau)dt = \frac{1}{2T}\int_{-T}^{T} dt \cdot e^{i\omega_0 t - i\omega_0(t+\tau)} \cdot \iint_{-\infty}^{\infty} dx \cdot dx' \cdot e^{(ik/D)(x^2 - x'^2)\cos^2\theta} \cdot e^{2ik\cos\theta[h(x,t) - h(x', t+\tau)]}. \tag{29}$$

If one now assumes that the dependence of the random variable h on x and t may be specified by a bivariate normal distribution, then (29) becomes:

$$\overline{P_Q}(\tau) = e^{-i\omega_0\tau}\iint d\xi dx \cdot e^{(ik/D)\cos^2\theta(2x\xi - \xi^2)} \cdot e^{-a[1-C(\xi,\tau)]}, \tag{30}$$

where

$$C(\xi, \tau) = \frac{1}{2T}\int_{-T}^{T} dt \cdot h(x, t) \cdot h(x + \xi, t + \tau), \tag{31}$$

$$\overline{P_Q}(\tau) = e^{-i\omega_0\tau} \cdot e^{-a[1-C(0,\tau)]}. \tag{31a}$$

But $C(0, \tau)$ is merely the autocorrelation function at a single point of the medium. If we represent this by a function of the form $e^{-\tau^2/2T_0^2}$, then we obtain for the frequency spectrum of the signal

$$P_Q(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{i\omega\tau}\overline{P_Q}(\tau)d\tau = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\tau \cdot e^{i(\omega-\omega_0)\tau} \cdot e^{-a}\sum_{n=0}^{\infty}\frac{a^n}{n!}e^{-n\tau^2/2T_0^2} \tag{32}$$

$$= e^{-a}\left\{\delta(\omega-\omega_0)+\frac{T_0}{\sqrt{2\pi}}\sum_{n=1}^{\infty}\frac{a^n}{n!\sqrt{n}}\right.$$
$$\left.\cdot e^{-[(\omega-\omega_0)^2/2n]T_0^2}.\right. \tag{32a}$$

We may thus draw the interesting conclusion that the frequency spectrum does not depend on the scale of coherence in the screen but only upon the depth of the irregularities and their time fluctuation, provided, of course, it is sufficiently accurate to neglect the distance attenuation factor. If this is not the case, then the integration of (30) requires a knowledge of the complete function $C(\xi, \tau)$. It is possible to assume a product of functions of ξ and τ individually in this case, and to proceed. Fig. 3 indicates, however, that if a' is used in place of a, the distance correction factor does not materially affect the total received power, indicating that (30) is generally sufficiently accurate as it stands, if a' is understood in place of a.

Two-Dimensional Surfaces

We now consider the random variable h to be a function of three independent variables x, y, and t. Taking account of the y-co-ordinate in the phase functions (2), (4) takes the form:

$$\overline{P}\sim\frac{1}{2T}\int_{-T}^{T}dt\cdot\iiiint_{-\infty}^{\infty}e^{(ik/\overline{D})\cos^2\theta(x^2-x'^2)+(ik/\overline{D})(y^2-y'^2)}$$
$$\cdot e^{2ik\cos\theta[h(x,y,t)-h(x',y',t)]}dxdx'dydy'. \tag{33}$$

If we transform variables through: $\xi=x'-x$, $\eta=y'-y$, then provided the distribution function of "$h(x, y, t)-h(x+\xi, y+\eta, t)$" is independent of x and y (homogeneous turbulence) we may integrate over x and y, obtaining delta functions of ξ and η. Integration over the latter two variables then yields the specular value for $\overline{P}$. To obtain the correlation pattern on the ground, we denote a displacement along the x direction by s, and along the y direction by v; then in analogy to (13) one obtains:

$$\overline{C}_Q(s, v) = \mathrm{Rl}\ e^{iks\sin\theta}\iiiint_{-\infty}^{\infty}e^{(2ik/\overline{D})\cos^2\theta\cdot x(\xi-s\overline{D}/2D_2)}$$
$$\cdot e^{(2ik/\overline{D})y(\eta-v\overline{D}/2D_2)}\cdot e^{-(ik/\overline{D})\cos^2\theta(\xi^2-s^2\overline{D}/2D_2)}$$
$$\cdot e^{-ik/\overline{D}(\eta^2-v^2\overline{D}/2D_2)}$$
$$\cdot\frac{1}{2T}\int_{-T}^{T}dt\cdot e^{2ik\cos\theta[h(x,y,t)-h(x+\xi,y+\eta,t)]}. \tag{34}$$

Denoting the time average integral by $f(\xi, \eta)$ one has:

$$\overline{C}_Q(s, v) = f\left(\frac{s\overline{D}}{2D_2}, \frac{v\overline{D}}{2D_2}\right)\cdot\cos\left[\frac{k\cos^2\theta}{2D_2}s^2\left(1-\frac{\overline{D}}{2D_2}\right)\right.$$
$$\left.+\frac{k}{2D_2}v^2\left(1-\frac{\overline{D}}{2D_2}\right)+ks\sin\theta\right]. \tag{34a}$$

In practice one might assume that the difference of the h's was distributed in accordance with a normal law, the variance depending only upon the total distance between points, independent of the direction (isotropic turbulence).

$$\frac{1}{2T}\int_{-T}^{T}dt\cdot h(x, y, t)\cdot h(x+\xi, y+\eta, t)$$
$$=\overline{h^2}\rho(\sqrt{\xi^2+\eta^2}). \tag{35}$$

Then

$$f(\xi, \eta) = e^{-a[1-\rho(\sqrt{\xi^2+\eta^2})]}. \tag{36}$$

To ascertain the angular power distribution, or the effect of the distance-attenuation factor, it is necessary to specify the form of ρ, and perform the integration with respect to ξ and η first. The vertical incidence ($\theta=0$) case will be used as illustration here. As a result of the symmetry we need consider only one displacement, say s. Then, transforming to polar co-ordinates:

$$\begin{aligned} r&=\sqrt{\xi^2+\eta^2}; & \xi&=r\cos\phi, & \eta&=r\sin\phi\\ R&=\sqrt{x^2+y^2}; & x&=R\cos\psi, & y&=R\sin\psi,\end{aligned} \tag{37}$$

and utilizing (36), (34) becomes:

$$\overline{C}_Q(s) = e^{(ik/D_2)s^2}\int_0^{2\pi}\int_0^{\infty}RdRd\psi\cdot e^{-2ik/\overline{D}(s\overline{D}/2D_2)R\cos\psi}$$
$$\cdot\int_0^{2\pi}\int_0^{\infty}rdrd\phi\cdot e^{(2ik/\overline{D})rR\cos(\phi-\psi)}$$
$$\cdot e^{-a[1-\rho(r)]}\cdot e^{-(ik/\overline{D})r^2}, \tag{38}$$

$$\overline{C}_Q(s) = e^{iks^2/2D_2}\int_0^{\infty}RdRJ_0\left(\frac{kRs}{D_2}\right)\int_0^{\infty}rdrJ_0\left(\frac{2krR}{\overline{D}}\right)$$
$$\cdot e^{-a[1-\rho(r)]-(ik/\overline{D})r^2}. \tag{38a}$$

One may note in passing that an application of the Fourier-Bessel transform theorem to (38a), yields (34a). It is now possible to define an angular power density:

$$p(R) = \int_0^{\infty}rdrJ_0\left(\frac{2krR}{\overline{D}}\right)\cdot e^{-(ik/\overline{D})r^2-a[1-\rho(r)]}, \tag{39}$$

and write:

$$\overline{C}_Q(s) = e^{iks^2/2D_2}\int_0^{\infty}p(R)\cdot J_0\left(\frac{kRs}{D_2}\right)\cdot RdR. \tag{40}$$

The Bessel function appearing here is to be contrasted with the exponential in (22), for the one-dimensional case. Physically, the slower rate of fall of J_0 is a consequence of the energy coming from directions transverse to the displacement s, and therefore producing no differential phase displacement between the two receiving points.

To evaluate $p(R)$, we must adopt a specific form for $\rho(r)$, say e^{-r^2/l^2}. Then:

$$p(R) = e^{-a}\sum_n\frac{a^n}{n!}\int_0^{\infty}rdr\cdot J_0\left(\frac{2krR}{\overline{D}}\right)e^{-nr^2/l^2-(ik/\overline{D})r^2}, \tag{41}$$

$$= e^{-a}\sum_{n=0}^{\infty} \frac{a^n}{n!} \frac{e^{-nR^2/l^2(1+[ng]^2)}}{\sqrt{1+(ng)^2}} \cdot \cos\left[\frac{kR^2/\overline{D}}{1+(ng)^2} + \arctan(ng)\right]. \quad (41a)$$

To take account of distance attenuation and obliquity, one now divides $p(R)$ by the factor

$$\left[1+\left(\frac{R}{D_1}\right)^2\right]^{3/2} \cdot \left[1+\left(\frac{R}{D_2}\right)^2\right]^{3/2}.$$

For the case where $g \gg 1$, and $D_1 = D_2$, one obtains:

$$p'(R) = \frac{e^{-a}}{\left[1+\left(\frac{R}{D}\right)^2\right]^3} \sum_{n=1}^{\infty} \frac{a^n}{n!} e^{-R^2k^2l^2/nD_2} + e^{-a}\cos\frac{kR^2}{D}, \quad (42)$$

$$\overline{P}_Q = \int_0^\infty dR\, R p'(R) = e^{-a}\left[1 + \sum_{n=1}^{\infty} \frac{a^n}{n!}\left\{\frac{(kl)^2}{2n} - \frac{(kl)^4}{2n^2} - \frac{e^{(kl)^2/2n}}{2\left(\frac{n}{k^2l^2}\right)^3} Ei\left(-\frac{k^2l^2}{n}\right)\right\}\right]. \quad (43)$$

For small "kl," one may apply a method similar to that which led to (21). In the present two-dimensional case, h can change with y as well as with x, so that one obtains:

$$a' = a/1 + \frac{4a}{(kl)^2}. \quad (44)$$

When this is combined with the results of (43) as has been done in Fig. 3(b), it appears that the ineffectiveness of the small scale turbulence, which is the type required to produce the wide-angle scattering, generally renders the distance correction unimportant. The frequency power spectrum is identical with the one-dimensional results, while the variance expressions have proved too complex to evaluate.

Conclusion

We have found it possible to obtain relatively simple expressions for various quantities associated with reflection of wave energy from a perfectly reflecting, randomly fluctuating surface by a straightforward application of the methods of physical optics to the statistical description of the surface. An extension of the range of validity of these methods has been attempted, to the case where more rigorous wave solutions are required. These results should consequently be viewed only as indicative of the behavior to be expected. In the remaining sections only the usual approximations of physical optics have been employed so that the results are deemed trustworthy within the limits of validity of the assumptions made initially.

A Preliminary Study of Fading of 100 Megacycle FM Signals*

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***Summary*—This paper describes a statistical study of the short-period fading of 100 megacycle FM signals over a mountainous propagation path under typical midday conditions. Signal intensities are found to be essentially Rayleigh distributed, while the fading increments follow a Gaussian distribution. Included are a statistical determination of the speed of fading and an effective wind velocity which may produce the fading, and a comparison of the statistically determined wind velocity with that measured by more conventional means.**

Introduction

INTEREST has been focused on long distance transmission of VHF signals during the past years and a tremendous amount of data has been taken of signal strength versus time, but the short-period anomalies in the signal strength have been largely neglected. Several theories have been postulated to explain these short-period fluctuations, but very few experiments have been made in an effort to support or refute them.

Using the Booker-Gordon[1] theory of scattering in the troposphere, and taking the statistical tools set forth by Ratcliffe[2] and used by Mitra,[3] an analysis has been made of the short-period fluctuations of signal strength on a propagation path between Pittsburgh, Pa.,[4] and State College, Pa.

Theory

Booker and Gordon state that the fading occurs through the mechanism of scattering from turbulent areas in the troposphere. The change in dielectric con-

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[1] H. G. Booker and W. E. Gordon, "A theory of scattering in the troposphere," Proc. I.R.E., vol. 38, pp. 401–412; April, 1950.
[2] J. A. Ratcliffe, "Diffraction from the ionosphere and the fadings of radio waves," *Nature*, vol. 162, pp. 9–11; July 3, 1948.
[3] S. N. Mitra, "Statistical analysis of fading of a single down coming wave from the ionosphere," *Jour. IEE*, vol. 96, part III, pp. 505–507; 1947.
[4] Two tests were made on other paths.

stant of the air within a turbulent area will cause a partial reflection of the incident electromagnetic wave. The energy of the electromagnetic wave is scattered in all directions. The energy that is directed toward the receiving antenna is of importance here.

With reference to Fig. 1 it can be seen that the lengths of the transmission paths resulting from scattering will cause a difference in phase between the various scattered signals at the receiver. If the scattering elements are stationary a steady signal will result, but if they are traveling with random velocities each scattering element will cause a continually varying phase shift

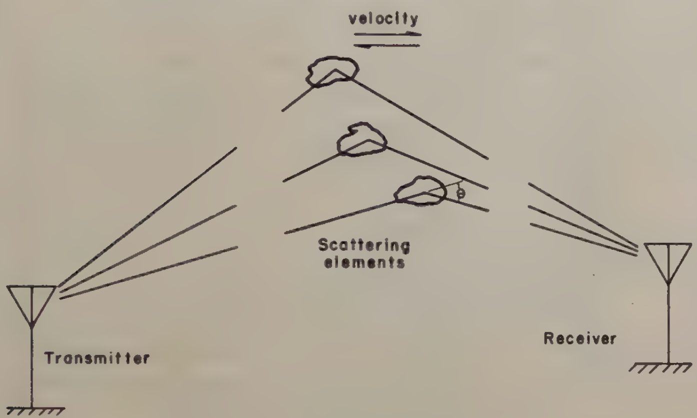

Fig. 1—Scattering of signals.

at the receiver, or in other words, a Doppler shift in each component of the frequency of the received signal. The frequency shift will be dependent upon the velocity of the scattering element and the interference of the signals from the various scattering elements will cause a randomly fading signal at the receiver.

If the scattering elements shown in Fig. 1 have no velocity, but are randomly changing in size and shape, they may cause a similar effect.

The magnitude of a signal that is fading randomly will follow a Rayleigh distribution[2]

$$P(R) = \frac{R}{\psi} \epsilon^{-R^2/2\psi} \tag{1}$$

where

$P(R)$ = Probability of amplitude R
R = Amplitude of received signal
$\psi = 2\overline{R}^2/\pi$
$\overline{R}$ = Mean amplitude of R

and the change of amplitude will follow a Gaussian distribution

$$P(\Delta R) = \frac{1}{\pi |\overline{\Delta R}|} \epsilon^{-\Delta R^2/\pi|\overline{\Delta R}|^2} \tag{2}$$

where

ΔR = Change in amplitude of R between samples
$P(\Delta R)$ = Probability of ΔR
$|\overline{\Delta R}|$ = Mean absolute change of R.

For a signal that follows the Rayleigh distribution of amplitude and a Gaussian distribution of change in amplitude, Ratcliffe defines a statistical quantity, called "speed of fading," given by the equation

$$S = \frac{|\overline{\Delta R}|}{\overline{R}_\tau} \tag{3}$$

where

S = Speed of fading
τ = Period between readings, relatively small with respect to $1/S$.

The speed of fading given by Booker and Gordon for a scattered signal in the troposphere is

$$S = \frac{2V \sin \theta/2}{\lambda}$$

where

V = RMS velocity of scattering element
λ = Wavelength of signal
θ = Angle through which scattering occurs.

Experiments

The receiving antenna was located on the roof of a building, therefore the position of effective ground was in doubt, and as a consequence the lobe structure of the receiving antennas could not be obtained. In experiments performed at the University of Texas and reported by Straiton, Metcalf, and Tolbert,[5] the angle of the arrival of the strongest signal on long tropospheric propagation paths was found to be the horizon; therefore, θ is taken as the sum of the two horizon angles of the transmitter and receiver measured above the line from transmitter to receiver. (This is the minimum possible value of θ.)

The 4/3 earth curvature was used to take into consideration the bending of the propagation path due to normal refraction.

Using (3) and (4), an equation for the RMS value of velocity of the scattering elements may be obtained in terms of values that can be taken from the fading record.

$$V = \frac{|\overline{\Delta R}| \lambda}{2\overline{R}_\tau \sin \theta/2}. \tag{5}$$

No direct correlation should be expected between the RMS value of wind velocity obtained above and the wind velocity measured with the aid of a pilot balloon. The velocity in these theories is an effective velocity composed of the combination of three physical effects: (1) Formation or decay of inhomogeneities in the atmosphere. (2) Movement of these inhomogeneous masses with respect to one another within a fixed boundary.

[5] A. W. Straiton, D. F. Metcalf, and C. W. Tolbert, "A study of tropospheric scattering of radio waves," Proc. I.R.E., vol. 39, pp. 613–648; June, 1951.

(3) Movement of the entire large air mass containing the inhomogeneities (a steady wind). This last has an effective velocity proportional to the wind velocity.[6] The wind velocity may be the same for two situations using the pilot balloon reading but at the same time the random motion may be different, which would result in an altered RMS value. If the wind is the most important of the three physical effects, then the speed of fading should show a more or less linear relation to the wind velocity, depending upon the amount of contamination due to the first two mentioned effects.

In another theoretical treatment of the subject, Rice[7] derives expressions which may be stated in the following way:

$$V_0 = (aU^2 + bV_n^2)^{1/2} \tag{6}$$

where

U = RMS turbulence velocity

V_0 = Effective velocity

V_n = Component of steady drift velocity perpendicular to path

a and b are constants, of the order of magnitude one.

Starting from somewhat different basic assumptions, Booker and Gordon[8] discuss correlation as functions of time and distance and arrive at results that may be stated as follows:

$$V_0 = (cU^2 + dV_n^2 + eV_p^2)^{1/2}$$

where

V_p = Component of wind velocity parallel to path

c, d, and e are constants. In an example they cite for illustrative purposes, d and c are of the same order of magnitude and e is much smaller.

From fading data at times when the winds aloft are practically zero, U can be inferred and compared with meteorological estimates.

Situations which may cause turbulence in the troposphere are thermals, shear between two masses of air moving relative to each other, and mixing occurring when atmospheric standing waves are present. The first two would have definite translational velocities while the latter would be relatively stationary.

If either of the first two forms of turbulence is causing the fading, the speed of fading obtained by the statistical analysis might have some correlation with the wind velocity. Turbulence caused by standing waves would probably give fading records which could not be correlated with tropospheric winds. The mountainous terrain, over which the propagation path under study is situated, may produce atmospheric standing waves. An alternate method of determining the wind velocity from fading records would be of assistance in determining whether standing waves exist, along with their associated type of turbulence.

Instrumentation

A crystal controlled double conversion superheterodyne receiver with an Esterline Angus recorder was used in obtaining the data for the statistical analysis. In cases of rapid fading the recorder was operated at 12 inches per minute chart speed. Using this chart speed the fading response of the receiver and recorder was checked[9] and the results are shown in Fig. 2.

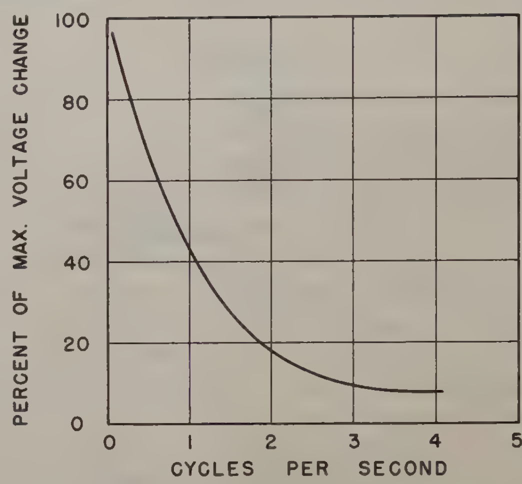

Fig. 2—Fading frequency response of receiving and recording system.

The period of time for which the data were taken in each case was held approximately constant at between 15 to 20 minutes. The recordings were usually taken at approximately the same hour during the day, about 1400.[10] The recordings were taken on days in which the fading was dissimilar in an attempt to get a good range in wind velocity. The charts were all analyzed on the same basis. A value of τ was chosen which gave from 300 to 1,600 readings on each chart; τ is always appreciably less than $1/S$.

Results

The majority of the fading tests showed a close approximation to a Rayleigh distribution of amplitude and a Gaussian distribution of amplitude changes. Results of a typical analysis are shown in Figs. 3 and 4. Solid lines represent Rayleigh and Gaussian distributions, respectively. An attempt was made when running the tests to get a spread of different fading speeds. The faster the fading the closer the results approach the theoretical. A plausible reason for this is that when fading was rapid, a much longer sample was taken in terms of the number of fading periods.

[6] R. E. McNicol, "The fading of radio waves of medium and high frequencies," *Jour. IEE*, vol. 96, part III, pp. 517–524; 1949.

[7] S. O. Rice, "Statistical fluctuations of radio field strength far beyond the horizon," Proc. I.R.E., vol. 41, pp. 274–281; February, 1953.

[8] Cornell University Research Report EE144.

[9] C. R. Ammerman, and R. L. Riddle, "A low-frequency modulator for receiver testing, *Electronics*, vol. 25, pp. 240, 244, 248; September, 1952.

[10] Tests 010, 5, and 15A were taken about 0800. Test 4z was taken at 2300.

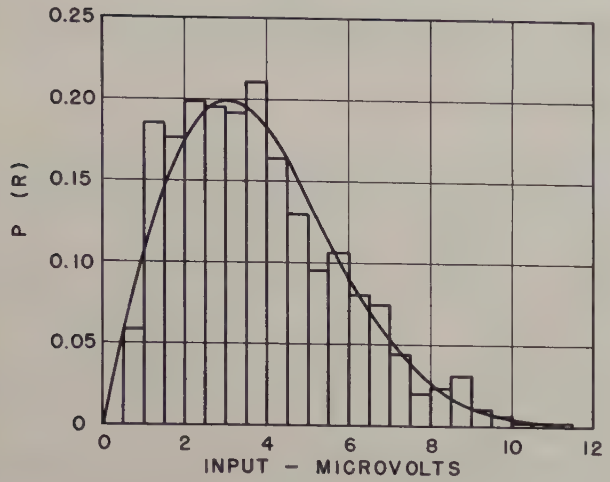

Fig. 3—Distribution of amplitude of fading signals, at 1400, November 14, 1951, sampled at 0.75 second intervals. Test 4 (curve is a Rayleigh distribution).

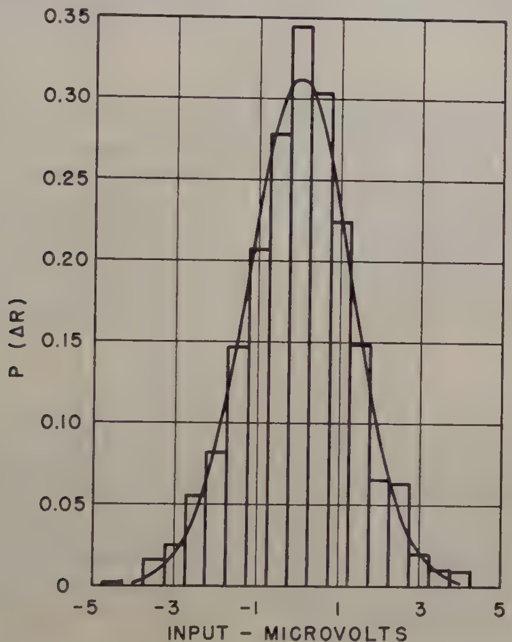

Fig. 4—Distribution of change in amplitude of fading signals, sampled at 0.75 second intervals, at 1400, November 14, 1951. Test 4 (curve is a Gaussian distribution).

The amplitude distribution of all data used is shown in Fig. 5. Numbers on the lines represent test numbers. Lines have been displaced vertically for ease of reading. A Rayleigh distribution would appear as a 45 degree line (such as dashed lines included for comparison). About 15% of the tests were discarded when an analysis showed their amplitude distributions deviated much more markedly from the Rayleigh distribution than the ones illustrated.

The effective values of velocity obtained by the statistical analysis were compared with the wind velocities recorded by the pilot balloon technique.[11] The values of wind velocity recorded at Pittsburgh are representative of the wind velocity over the propagation path. Air

[11] Winds aloft data from Pittsburgh or station nearest the propagation path reporting applicable data.

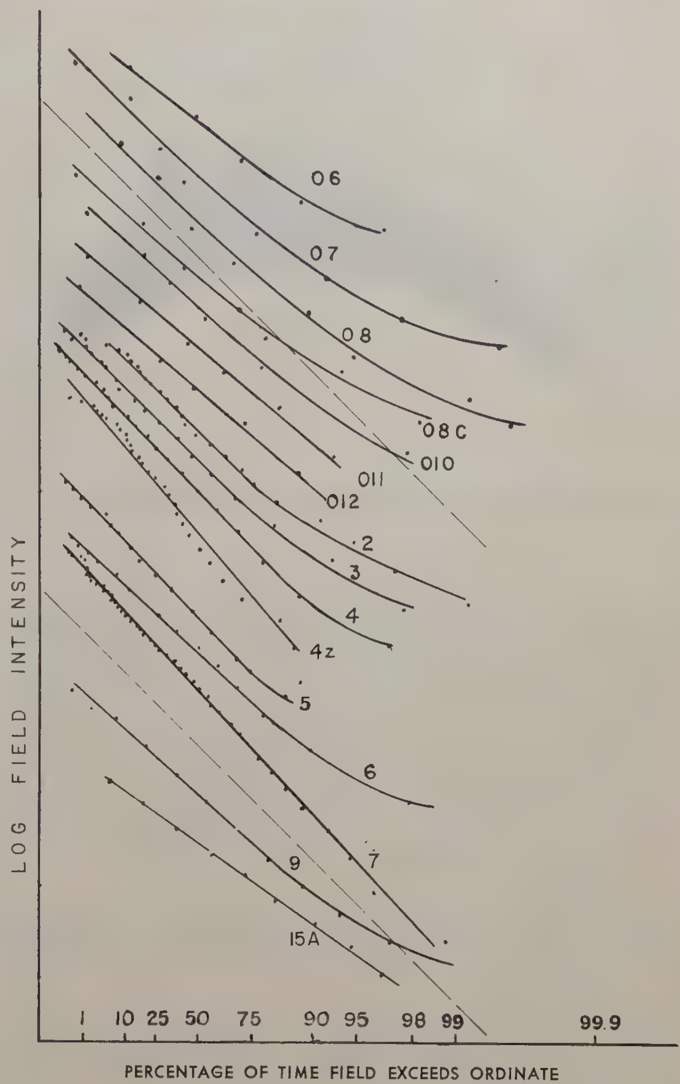

Fig. 5—Distribution of amplitude of fading signals. (A Rayleigh distribution forms a 45 degree line. Dashed lines at 45 degrees for comparison.)

moves in large masses and therefore the upper air wind velocities are fairly constant over wide areas. A closer approximation may be obtained of the wind velocity at a given point by interpolation between weather stations, or by the plotting of a wind flow chart. This was not performed in this particular instance as it was considered an extra refinement which would add little to the end results. The value of wind velocity at the scattering volume is probably the least accurately determined quantity of the tests.

In tests 3, 4, and 7, in particular, the value obtained for velocity V_n is very much in doubt because of frontal activity near the scatter volume.

The volume of the troposphere that produces scattering on this propagation path lies approximately 39 miles west of State College and is at an altitude of approximately 3,800 feet. These data were obtained from the profile map shown in Fig. 6.[12]

[12] Test 15A is for a path from Easton, Pa., to State College, Pa Test 08C is for a path from Olean, N. Y., to State College, Pa. In these tests appropriate values for θ and V_n were obtained.

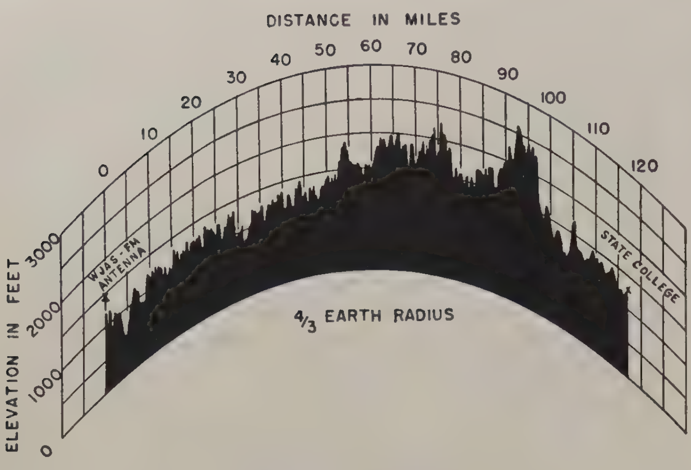

Fig. 6—Profile of terrain between transmitter in Pittsburgh, Pa., and receiver in State College, Pa. (Using 4/3 earth radius.)

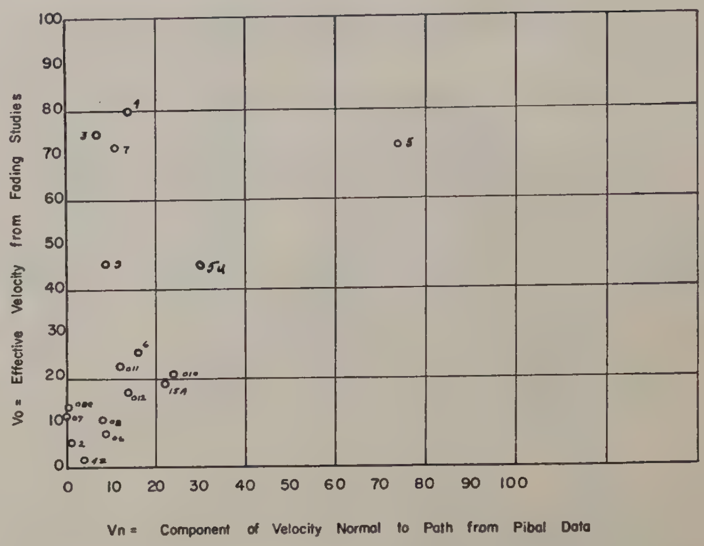

Fig. 7—Effective velocity calculated from fading data compared with normal component of velocity as measured with pilot balloon.

The component of the wind velocities normal to the propagation path is plotted against the value obtained by the statistical analysis and may be seen in Fig. 7. The following factors relating to Fig. 7 are emphasized:

(a) Wind velocity is measured at Pittsburgh.[12] Scatter volume is about 80 miles away.

(b) Altitude of scatter volume is from about 3,800 feet up. Wind velocity was measured at 4,000 feet, if possible, otherwise nearest available figure.[11]

(c) Angle of scatter was chosen based on 3,800 feet altitude.[12] (Intersection of horizon lines at 4/3 earth radius.)

(d) Winds are measured at 0400, 1000, 1600, and 2400. Linear interpolation for time of wind data was used.

(e) If the wind (normal component) is the only important agent, theory predicts a linear relationship for Fig. 7.

Conclusion

With the aid of Ratcliffe's theory on ionospheric fading, the model of the troposphere postulated by Booker and Gordon is strengthened. A useful by-product of this investigation may be a method of continuously measuring the wind velocities of air masses in the troposphere, or determining the extent to which turbulence is present. More work should be done to obtain more refined results.

Acknowledgment

The authors wish to express their appreciation for the help of Mr. John Sherrod, formerly of the Department of Meteorology of the Pennsylvania State College, now with the Library of Congress, Washington, D. C. The data were obtained during the work on a contract on VHF Field Intensity Measurements, with the National Bureau of Standards.

VHF Field Intensities in the Diffraction Zone

R. N. GHOSE AND W. G. ALBRIGHT†

Summary—For the past few years there has been a general interest in the mechanism of propagation of vhf waves in the diffraction zone. Several theories have been advanced which attempt to describe the phenomena quantitatively, however, most calculated results from these theories have yielded signal strengths several decibels below those obtained experimentally.

This analysis follows the general procedure of obtaining a solution to the wave equation, subject to the appropriate boundary conditions. In obtaining this solution, it is necessary to make some assumption regarding the distribution of the refractive index as a function of the height above the surface of the earth. The essential difference between this solution and those previously presented is the form of this variation. An exponential form which approaches unity at large heights has been chosen. Using this distribution, an expression for the field strength as a function of separation between the transmitting and receiving antennas and the height of the antennas above the earth is obtained and signal strengths are calculated for comparison with measured values.

General Wave Equation

THE ELECTROMAGNETIC FIELD from a transmitter can be determined from a scalar wave equation of the form[1]

$$\nabla^2\psi + k^2\mu^2\psi = 0$$

where

$$k = \frac{2\pi}{\lambda} \qquad k^2\mu^2 = \frac{\epsilon\omega^2 - j\sigma\omega}{c^2}$$

and $r\psi(r, \theta, \phi)$ is the magnitude of the Hertzian potential in the radial direction. A time variation of $e^{j\omega t}$ is assumed.

For a free space region this equation can be reduced to the form:

$$\nabla^2\psi + k^2n^2\psi = 0 \tag{1}$$

where n depends on meteorological factors and is a function of the height above the surface of the earth (h).

This equation can be expanded in spherical co-ordinates and separated into two independent equations by assuming a solution of the form $\psi = P(\theta)U(r)$,

$$\frac{d^2U}{dr^2} + \frac{2}{r}\frac{dU}{dr} + \left[k^2n^2 - a^2\frac{K_m^2}{r^2}\right]U = 0 \tag{2}$$

$$\frac{d^2P}{d\theta^2} + \cot\theta\frac{dP}{d\theta} + [a^2K_m^2]P = 0 \tag{3}$$

where $a^2K_m^2$ is the separation constant.

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[1] P. Frank and R. Mises, "Die Differential und Integralgleichungen der Mechanik und Physik," Mary Rosenberg, New York, p. 871, 1943.

Distance Attenuation Factor

The solution of (3) yields the distance attenuation factor. Substituting for θ, the angle subtended at the center of the earth by the radii through the antennas, the expression $\theta = \rho/a$ (a being the radius of the earth and ρ the distance between the transmitter and receiver) this equation becomes

$$\frac{d^2P}{d\rho} + \frac{\cot(\rho/a)}{a} + \frac{dP}{d\rho} + K_m^2P = 0. \tag{4}$$

The solution of this equation[2] is well known and takes the form

$$P = AP_\eta(\cos\rho/a) + BP_\eta(-\cos\rho/a).$$

The second term of this expression is eliminated from the solution by the boundary condition that the only singularity that can exist is at the transmitting antenna. $P_\eta(-\cos\rho/a)$ introduces a singularity for all values of r when θ is zero, thus $B=0$.

P_η is the Legendre function of degree η where

$$\eta = -1/2 \pm \sqrt{1/4 + K_m^2a^2}.$$

If the diffraction zone only is to be considered, the asymptotic solution can be expressed as:

$$P_\eta(\cos\rho/a)$$

$$= K'\left[\frac{2}{\pi(\eta+1)\sin\rho/a}\right]^{1/2}\cos[(\eta+1/2)\rho/a - \pi/4]$$

$$\approx K'\left[\frac{2}{\pi K_m a\sin\rho/a}\right]^{1/2}e^{-j(K_m\rho-\pi/4)} \tag{5}$$

where K' is an arbitrary constant.

Height Gain Factor

The solution of (2) yields an expression for the variation of signal strength with height. Using the transformation $Q = U(h+a) = U(r)$ where h is the height above the earth and a is the radius of the earth, the equation

$$\frac{d^2Q}{dh^2} + \left[k^2n^2 - \frac{a^2K_m^2}{(h+a)^2}\right]Q = 0 \tag{6}$$

is obtained.

The distribution of the refractive index is to be substituted into this expression for n. This function is determined by atmospheric conditions and in general will be quite complex in form. However, under stable uni-

[2] E. C. Jordan, "Electromagnetic Waves and Radiating Systems," Prentice-Hall, Inc., New York, N. Y., p. 487; 1950.

form meteorological conditions a reasonable assumption for this distribution would have the form

$$n - 1 = (n_0 - 1)e^{-ch}$$

$$\frac{dn}{dh} = -c(n_0 - 1)e^{-ch}$$

and

$$\frac{dn}{dh} = -c(n_0 - 1),$$

at the earth's surface, where n_0 is the index of refraction at the surface of the earth.

This distribution fits all of the required boundary conditions, namely that the index of refraction must be an analytic function of height, must asymptotically approach unity at a very large height, and must have a linear gradient at and near the earth's surface. This distribution of the index of refraction corresponds to that suggested by Miss A. C. Stickland[3] as a result of measurement taken in England.

Substituting the assumed distribution of the refractive index into (6) yields

$$\frac{d^2Q}{dh^2} + \left[k^2(1 + (n_0 - 1)e^{-ch})^2 - \frac{K_m{}^2a^2}{(a + h)^2}\right] Q = 0. \quad (7)$$

This equation can be written in the form

$$\frac{d^2Q}{dh^2} + [Y(h)]Q = 0$$

where

$$Y(h) = k^2 + 2k^2(n_0 - 1)e^{-ch} + k^2(n_0 - 1)^2e^{-2ch} - \frac{K_m{}^2a^2}{(a + h)^2}. \quad (8)$$

It can be shown from the expansion of $Y(h)$ that the distribution with respect to height may be approximated by

$$Y(h) \approx \overline{Y}(h) = k^2n_0{}^2 - ack^2(n_0 - 1)n_0 + [an_0ck^2(n_0 - 1) - K_m{}^2]e^{-2h/a}. \quad (9)$$

The approximations involved in obtaining (9) are such that $\overline{Y}(h)$ not only resembles $Y(h)$ at low heights and agrees with the first order term identically, but also retains a bounded value at infinity. The deviation of $\overline{Y}(h)$ from $Y(h)$ is very small even at large heights, the maximum being of the order of $0.25k^2$ at infinity. This assumed distribution of $\overline{Y}(h)$ avoids the difficulty experienced in the Eckersley-Millington profile.

Making use of the following transformations

$$x = -h/a + t \qquad t = \log_e [ak\sqrt{\alpha a n_0 - K_m{}^2/k^2}]$$

$$\lambda = a^2k^2(n_0{}^2 - \alpha a n_0)$$

where α is the gradient of the refractive index, dn/dh at $h=0$; (7) can be written in the form of

$$\frac{d^2Q}{dx^2} + [e^{2x} + \lambda]Q = 0. \quad (10)$$

The general solution[4] for this equation is

$$Q = RJ_{-r}(e^x) + SJ_{+r}(e^x) \quad (11)$$

where

$$r = j\sqrt{\lambda}.$$

The second term of this equation represents an incoming wave and thus is not of interest in this solution. The height-gain function can now be written as

$$U(h) = \frac{R}{h + a} J_{-r}[ak\sqrt{\alpha a n_0 - K_m{}^2/k^2}\, e^{-h/a}] \quad (12)$$

where $\alpha = (n_0 - 1)c$ and R is a constant to be evaluated.

Total Field in the Diffraction Zone

It has been shown by Bremmer[5] that the components of the electric field for horizontal polarization can be determined from

$$E\phi = \frac{\omega}{jc}\frac{\partial\psi}{\partial\theta} = -jkU\frac{dP}{d\theta} \text{ (cgs electrostatic units)}$$

$$= -j\omega U \frac{dP}{d\theta} \times 10^{-6} \text{ (MKS units).}$$

The factor $|dP/d\theta|$ can be reduced to

$$\left|\frac{dP}{d\theta}\right| \approx a\sqrt{\frac{2K_m}{\pi\rho}}\, e^{-\mathrm{Im}(K_m\rho)}. \quad (13)$$

$E\phi$ can now be written as

$$|E\phi| = a\sqrt{\frac{2K_m}{\pi\rho}}\,\omega|U|\, e^{-\mathrm{Im}(K_m\rho)} \times 10^{-6}. \quad (14)$$

It can be shown following the procedure adapted by Pekeris and Ament[6] that the total field F in the diffraction zone can be expressed as

$$|F| = \omega a\sqrt{\frac{2K_m}{\pi\rho}}\,|U(h_1)||U(h_2)|\, e^{-\mathrm{Im}(K_m\rho)} \times 10^{-6}$$

where h_1 and h_2 are the heights of the transmitting and receiving antennas.

If E_{FS} denotes the free space field, the total field in terms of the free space field can be written as[7]

[3] A. C. Stickland, "Meteorological Factors in Radio-Wave Propagation," Refraction in the Lower Atmosphere and its Application to the Propagation of Radio Waves, The Physical Society and the Royal Meteorological Society, London; 1946.

[4] E. C. Titchmarsh, "Eigenfunction Expansions Associated with Second-order Differential Equations," Oxford, The Clarendon Press, p. 84; 1946.

[5] H. Bremmer, "Terrestrial Radio Waves," Elsevier Publishing Co., p. 14.

[6] C. L. Pekeris and W. S. Ament, "Characteristic of the first normal mode in the problem of microwaves," *Philosophical Magazine*, vol. 38, p. 801; November, 1947.

[7] D. E. Kerr, "Propagation of Short Radio Waves," McGraw-Hill Book Co., New York, N. Y.; 1951.

$$|F| = E_{FS}\left[\omega a\sqrt{\frac{2K_m}{\pi}}\sqrt{d}\,e^{-\mathrm{Im}(K_m\rho)}\,|U(h_1)|\,|U(h_2)|\right] \cdot \frac{10^{-6}}{\sqrt{5280}}, \tag{15}$$

where d is the distance between the transmitter and receiver in miles.

Calculation of Eigenvalues

The eigenvalues can be determined from the boundary condition that at the surface of the earth $U(h)=0$. From (12)

$$U(0) = \frac{R}{a} J_r(ak\sqrt{a\alpha n_0 - K_m^2/k^2}) = 0$$

$$\therefore \quad J_r(ak\sqrt{a\alpha n_0 - K_m^2/k^2}) = 0.$$

An approximate value of the lowest zero of the Bessel function can be obtained by the extension by analytic continuation of the method used by Watson.[8]

$$Z_{h=0} = ak\sqrt{a\alpha n_0 - K_m^2/k^2} = r + r^{1/3}(1.85576) + O(r^{-1/3})$$

where

$$r = jak\sqrt{a\alpha n_0 - n_0^2}.$$

Squaring both sides we obtain:

$$a^2k^2\left(a\alpha n_0 - \frac{K_m^2}{k^2}\right) = -a^2k^2(n_0^2 - a\alpha n_0) + 1.85576^2 r^{2/3}(0.5 + j0.867) + 2(1.85576)r^{4/3}(-0.5 - j0.867)$$

$$\mathrm{Im}\left(\frac{K_m^2}{k^2}\right) \approx \frac{+1.732(1.85576)(n_0^2 - a\alpha n_0)^{2/3}}{(ak)^{2/3}}. \tag{16}$$

Assuming

$$K_m = p + jq \qquad \mathrm{Im}\,(K_m^2) = 2pq$$

but $p \approx kn_0$

$$\mathrm{Im}\left(\frac{K_m^2}{k^2}\right) \approx \frac{2qn_0}{k} = +2\left[\frac{(n_0^2 - a\alpha n_0)^{2/3}}{(ak)^{2/3}}\right]0.92788 \times 1.7321$$

$$\therefore \mathrm{Im}\,(K_m) = q \approx +k^{1/3}\frac{(n_0 - a\alpha n_0)^{2/3}}{a^{2/3}n_0}0.92788 \times 1.7321. \tag{17}$$

The real value of K_m corresponding to the lowest attenuation can be found from (16) and the imaginary value from (17).

[8] G. N. Watson, "A Treatise on the Theory of Bessel Functions," Cambridge University Press, London, Eng., p. 521; 1922.

Normalization of Height-Gain Function

In order to complete the solution for the height-gain function (12) the constant R must be determined. Following the normalization procedure of Pekeris and Ament[6] one obtains

$$\left|\left(\frac{dQ}{dh}\right)^2_{h=0}\right| \approx 1$$

$$Q = RJ_r\left(ak\sqrt{a\alpha n_0 - \frac{K_m^2}{k^2}e^{-h/a}}\right)$$

$$\frac{dQ}{dh} = -Rk\sqrt{a\alpha n_0 - \frac{K_m^2}{k^2}} \cdot \left\{\left[\frac{r}{ak\sqrt{a\alpha n_0 - \frac{K_m^2}{k^2}e^{-h/a}}}\right] \cdot J_r\left(ak\sqrt{a\alpha n_0 - \frac{K_m^2}{k^2}e^{-h/a}}\right) - J_{r+1}\left(ak\sqrt{a\alpha n_0 - \frac{K_m^2}{k^2}e^{-h/a}}\right)\right\} e^{-h/a}$$

$$\left|\left(\frac{dQ}{dh}\right)^2_{h=0}\right| = \left|R^2k^2\left(a\alpha n_0 - \frac{K_m^2}{k^2}\right) \cdot J_{r+1}^2\left(ak\sqrt{a\alpha n_0 - \frac{K_m^2}{k^2}}\right)\right| \approx 1$$

$$\therefore \quad |R| = \left|\frac{1}{k\sqrt{a\alpha n_0 - \frac{K_m^2}{k^2}}\,J_{r+1}\left(ak\sqrt{a\alpha n_0 - \frac{K_m^2}{k^2}}\right)}\right| \tag{18}$$

$$|U(h)| = \frac{1}{\left|k\sqrt{a\alpha n_0 - \frac{K_m^2}{k^2}}(h+a)\right.} \cdot \left[\frac{J_r(ak\sqrt{a\alpha n_0 - K_m^2/k^2\ e^{-h/a}})}{J_{r+1}(ak\sqrt{a\alpha n_0 - K_m^2/k^2})}\right]\Bigg|. \tag{19}$$

Eq. (19) can also be expressed as

$$|U(h)| = \frac{1}{h+a}\left|\frac{Q(h)}{\left|\frac{dQ}{dh}\right|_{h=0}}\right| \tag{20}$$

and for numerical computations $U(h)$ can be computed for low heights by assuming a series solution for $Q(h)$ to satisfy (7).

Numerical Results

The total field in the diffraction zone can now be evaluated from (15), (16), (17), and (20) for a given set of parameters. The free space field intensity (E_{FS}) appearing in (15) can be determined from the expression

$$E_{FS} = \frac{186.3\sqrt{KW}}{d}\ (\text{mV/meter})$$

where d is the distance between the transmitter and receiver in miles and KW is the effective radiated power of the transmitter in kilowatts. Values of K_m can be determined from (16) and (17). A typical value of K_m for

$$\alpha = 1.2 \times 10^{-8}/\text{foot}$$
$$n_0 - 1 = 350 \times 10^{-6}$$
$$\text{Frequency} = 191.75 \text{ mc/sec}$$
$$K_m = 1.23542 + j1.687711 \times 10^{-5}$$

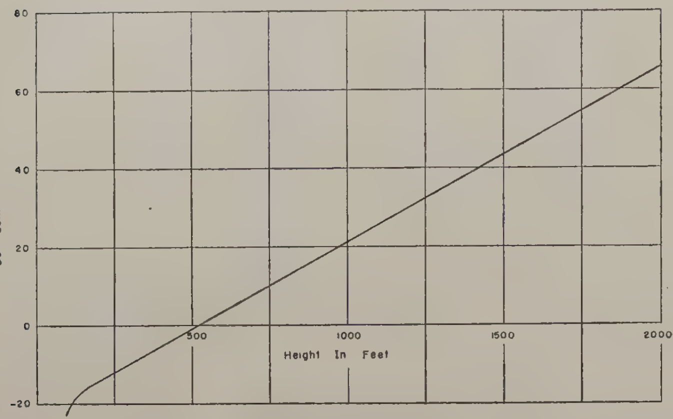

Fig. 1—Height-gain function. (Relative.)
$n_0 - 1 = 300 \times 10^{-6}$ WGN-TV (191.75 mc)
$dn/dh = -1.2 \times 10^{-8}$

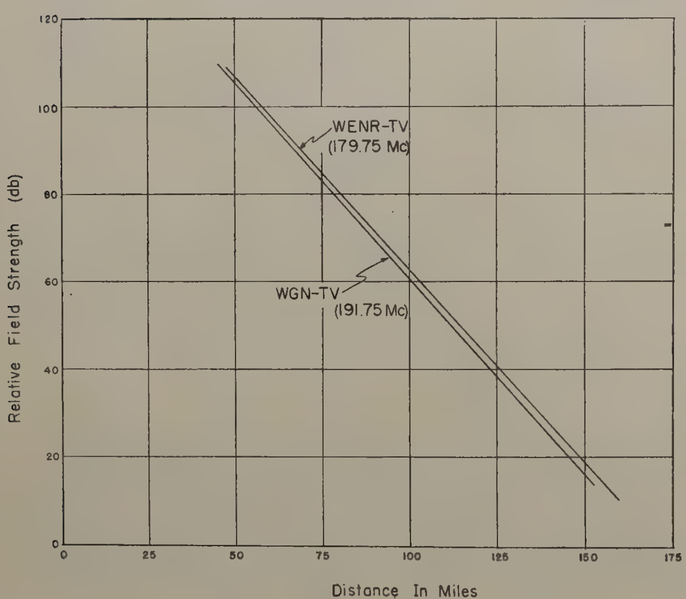

Fig. 2—Relative field strength vs. distance.
$(n_0 - 1) = 300 \times 10^{-6}$ $dn/dh = 1.2 \times 10^{-8}$/foot

Graphs illustrating the relative variation of the height-gain function and distance-attenuation function are shown in Figs. 1 and 2. These figures are included to indicate the type of variation to be expected and are not intended to be used obtaining signal strengths.

Fig. 3 is a plot of field intensity as calculated from (15) for a given set of parameters for which signal strength recordings are available. Included on this figure are points representing measured signal strengths as obtained in Urbana, Illinois, with the signal originating in Chicago, Illinois, and propagating over essentially a smooth earth for a distance of 127 miles. Located near

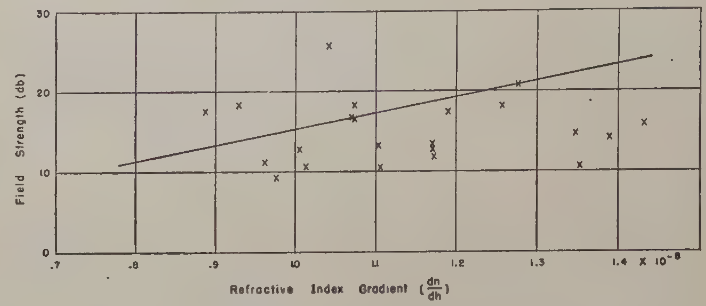

Fig. 3—Calculated and measured field strengths vs. refractive index gradient.
$n_0 - 1 = 300 \times 10^{-6}$ WGN-TV (191.75 mc)
$h_1 = 1{,}182$ feet $h_2 = 865$ feet

this propagation path are two Radio Sound stations, one in Joliet, and the other in Rantoul, Illinois. Signal strengths indicated were selected for periods when both meteorological stations indicated linear distributions of refractive index with nearly identical gradients. An examination of meteorological data for a period of one year yielded very few periods satisfying the condition imposed on the refractive index distributions and thus a limited number of signal strengths could be used for comparison. The fact that the values of signal strengths indicated lie within a few decibels of the calculated values is encouraging. The deviation of the measured field intensities from the theoretical values can be explained by the fact that the atmosphere is considered to be radially stratified in the derivation; however, this condition is not physically realizable. Since the deviation of measured and calculated signal intensities is rather small, a reasonable approximation of vhf field intensities can be obtained from the analysis for the particular set of physical conditions and terrain considered.

Acknowledgment

The guidance of Dr. E. C. Jordan of the University of Illinois is gratefully acknowledged. Further, it is desired to acknowledge the support of the National Bureau of Standards for which signal strength recordings have been made for the past several years. This recording program not only created an interest in the subject but also furnished the data used for comparison with the calculated results.

Mutual Impedance of Stacked Rhombic Antennas*

J. G. CHANEY†, SENIOR MEMBER, IRE

THE mutual impedance of two rhombic antennas[1] is given by

$$j\kappa Z_{21}/30 = \oint_1 \oint_2 e(r_{12}) \left[\frac{\partial^2}{\partial s_1 \partial s_2} - k^2 \cos\theta(s_1, s_2) \right] g(ks_1, ks_2) ds_2 ds_1 \quad (1)$$

in which

$k = 2\pi/\lambda$

s_1, s_2 = arc length co-ordinates

r_{12} = distance between positions on different wires

$\theta(s_1, s_2)$ = difference of directions of elements of arc length

$g(ks_1, ks_2) = R_e[f_1(ks_1)^* f_2(ks_2)]$

$f(ks)$ = normalized current distribution function

$e(r_{12}) = r_{12}^{-1} \exp(-jkr_{12})$

R_e = real part to be taken

$*$ = complex conjugate to be taken

Consider two stacked identical rhombic antennas with each leg being l meters in length and with each antenna having a vertex angle of 2α at the driving point. Let

$$L = [h^2 + l^2]^{1/2}, \qquad M = [h^2 + (2l \sin\alpha)^2]^{1/2},$$
$$N = [h^2 + (2l \cos\alpha)^2]^{1/2}, \qquad a = kh \tan\alpha$$

in which h is the height of one antenna above the other. Then, in terms of the functions

$$Si(x) = \int_0^x \sin t/t \, dt, \qquad Ci(x) = \int_\infty^x \cos t/t \, dt$$

$$Sia(x) = \int_0^x \frac{t \sin t}{t^2 + a^2} dt, \qquad Cia(x) = \int_\infty^x \frac{t \cos t}{t^2 + a^2} dt$$

(1) may be integrated to give

$$\begin{aligned} Z_m/120 = {} & 4[Ci(kh) - Cia(kh)] + 2[2Ciak(L - l) \\ & - Cik(L - l)] + 2[2Ciak(L + l) - Cik(L + l)] \\ & - 2Cia(kM) - Ciak(2l + N) - Ciak(2l - N) \\ & + \cos(2kl \sin^2\alpha)[Cik(N + 2l \cos^2\alpha) \\ & + Cik(N - 2l \cos^2\alpha) + Cik(M + 2l \sin^2\alpha) \\ & + Cik(M - 2l \sin^2\alpha) - 2Cik(L + l \cos 2\alpha) \\ & - 2Cik(L - l \cos_2 \alpha)] \\ & - \sin(2kl \sin^2\alpha)[Sik(N + 2l \cos^2\alpha) \\ & - Sik(N - 2l \cos^2\alpha) - Sik(M + 2l \sin^2\alpha) \\ & + Sik(M - 2l \sin^2\alpha) - 2Sik(L + l \cos 2\alpha) \\ & + 2Sik(L - l \cos 2\alpha)] \\ & + j\{4[Sia(kh) - Si(kh)] + Siak(N + 2l) \\ & + Siak(N - 2l) + 2Sia(kM) \\ & - 2[2Siak(L + l) - Sik(L + l)] \\ & - 2[2Siak(L - l) - Sik(L - l)] \\ & - \cos(2kl \sin^2\alpha)[Sik(N + 2l \cos^2\alpha) \\ & + Sik(N - 2l \cos^2\alpha) + Sik(M - 2l \sin^2\alpha) \\ & + Sik(M + 2l \sin^2\alpha) - 2Sik(L + l \cos 2\alpha) \\ & - 2Sik(L - l \cos 2\alpha)] \\ & - \sin(2kl \sin^2\alpha)[Cik(N + 2l \cos^2\alpha) \\ & - Cik(N - 2l \cos^2\alpha) + Cik(M - 2l \sin^2\alpha) \\ & - Cik(M + 2l \sin^2\alpha) - 2Cik(L + l \cos 2\alpha) \\ & + 2Cik(L - l \cos 2\alpha)]\}. \end{aligned} \quad (2)$$

The functions $Sia(x)$ and $Cia(x)$ are called the associated sine integral and cosine integral functions, respectively. They have been tabulated[2] by tenths from zero to 3.1 for both variables. They are being tabulated further by tenths for a ranging from one to twenty and by units for x from one to 200. Some of the properties of these functions are considered in the report being abstracted.

Upon passing to the limit as the height approaches zero, formula (2) reduces to the formula for the self impedance of a rhombic antenna.[3]

* Abstract of U. S. Naval Postgrad. School, Tech. Rept. no. 8, Feb., 1953.

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[1] J. G. Chaney, "Simplification for mutual impedance of certain antennas," *Jour. Appl. Phys.*, vol. 24, no. 6, p. 747; 1953.

[2] W. E. Bleick, "Tables of associated sine and cosine integral functions and of related complex valued functions," U. S. Naval Postgrad. School Tech. Rpt. no. 10, May, 1953.

[3] J. G. Chaney, "Free space radiation impedance of rhombic antenna," *Jour. Appl. Phys.*, vol. 24, no. 5, p. 536; 1953.

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